Dependency Graph Parsing as Sequence Labeling

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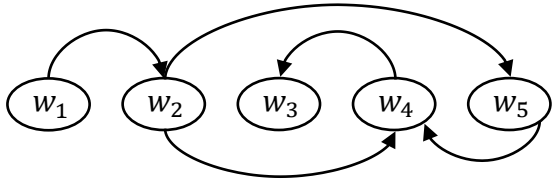
Motivation: Approaches in Dependency Graph Parsing

Dependency **Graph** Parsing…

- Sentence: $W = (w_1, \dots, w_n)$.
- Arcs: $A = \{(h, d): w_h \rightarrow w_d, h \neq d\}$.



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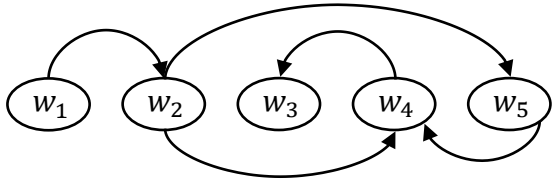

It's **not** Dependency *Tree* Parsing.

- Multiple roots and heads per node.
- Optionally non-connected.
- Cycles are allowed.

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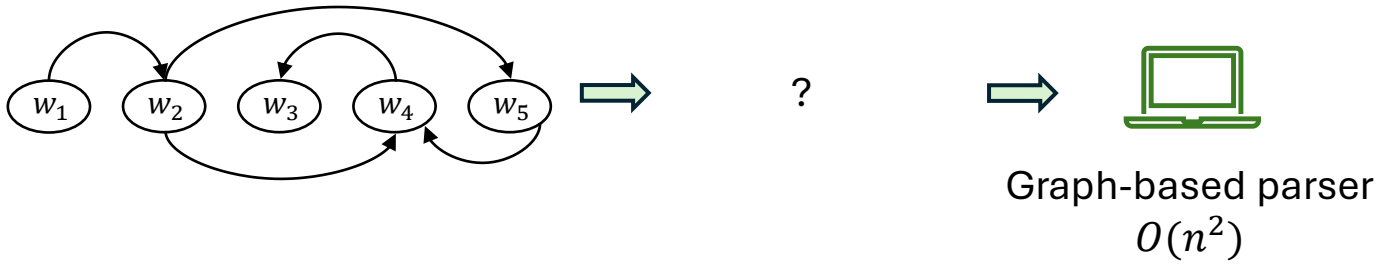

Graph-based parser
$O(n^2)$

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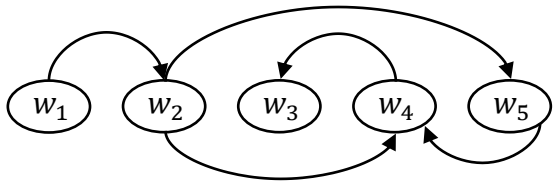

Adjacency matrix

0	0	0	0	0
1	0	0	0	0
0	0	0	1	0
0	1	0	0	1
0	1	0	0	0

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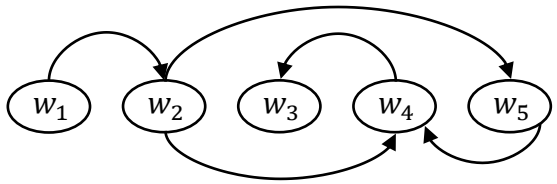

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Predicted scores

0	0.2	0.4	0.3	0.1
0.7	0	0	0.5	0.1
0.2	0	0	0.6	0
0.1	0.5	0.1	0	0.8
0.1	0.1	0.3	0.4	0

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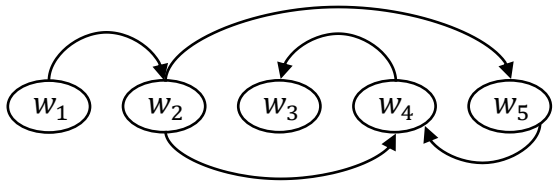

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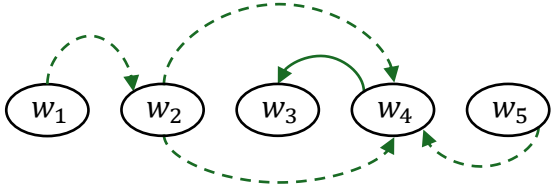

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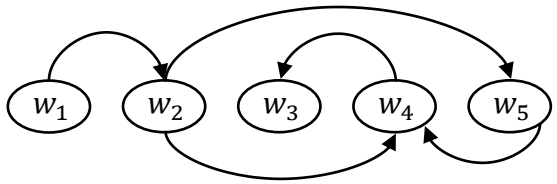

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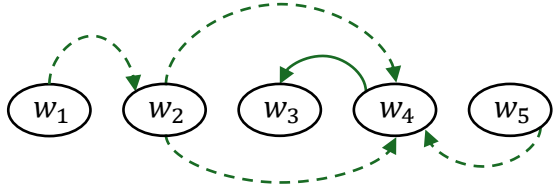

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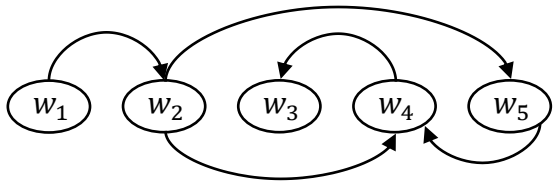

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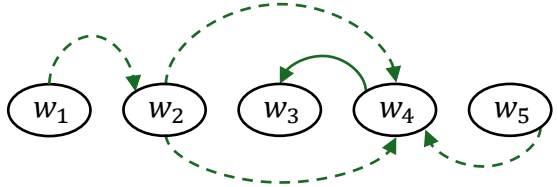

Encoding

$\ell_1 \quad \ell_2 \quad \ell_3 \quad \ell_4 \quad \ell_5$

Encoded labels

… as **Sequence Labeling**.

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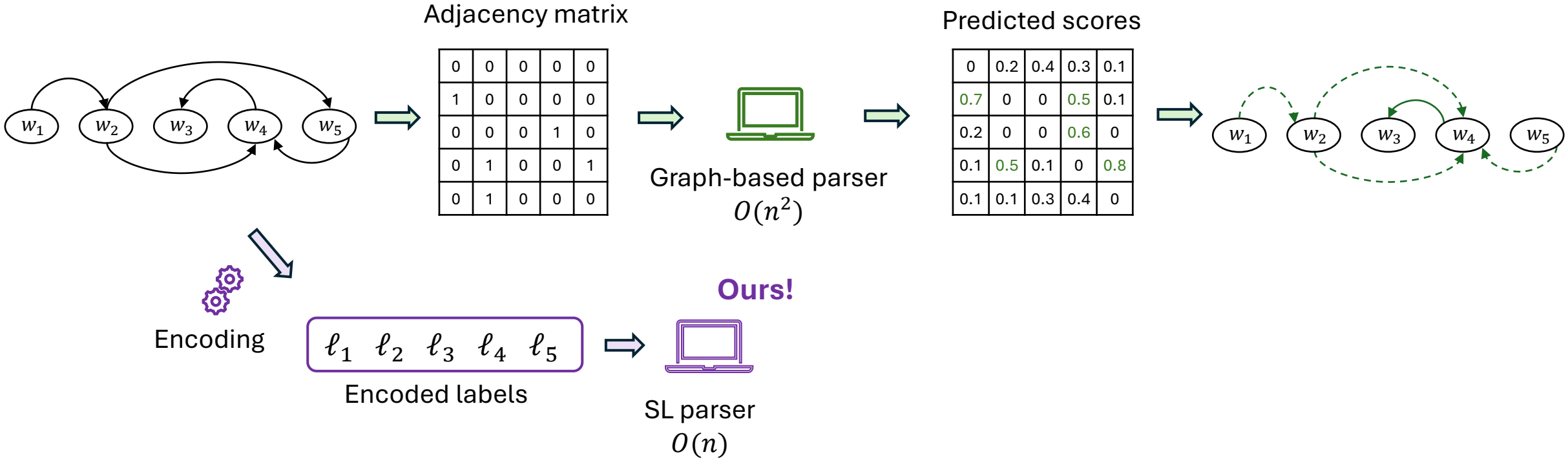

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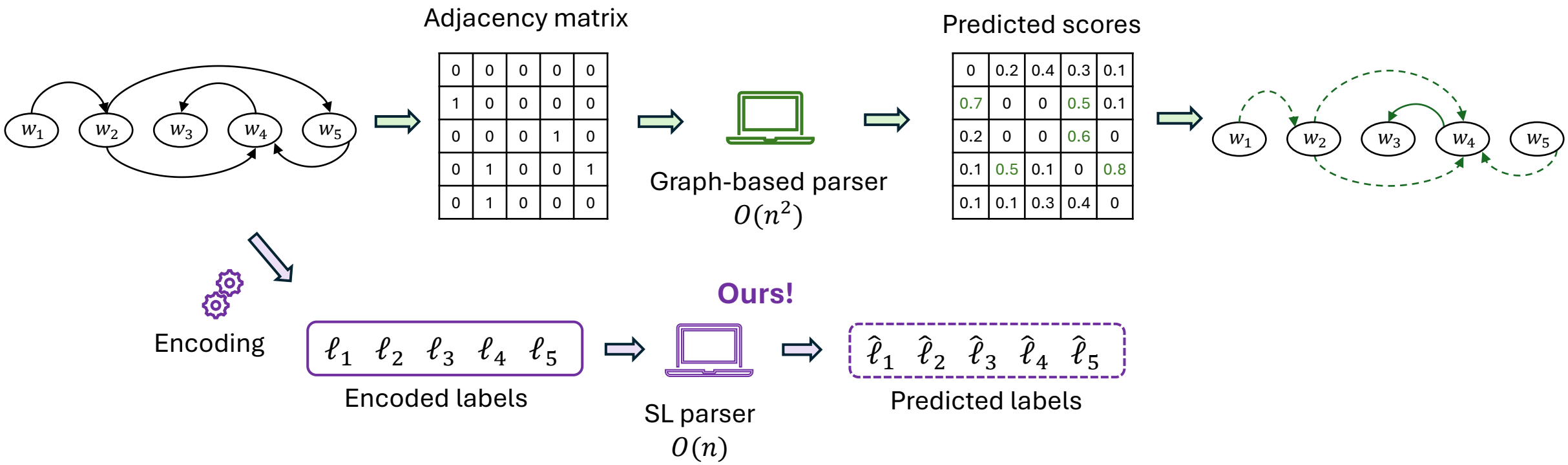

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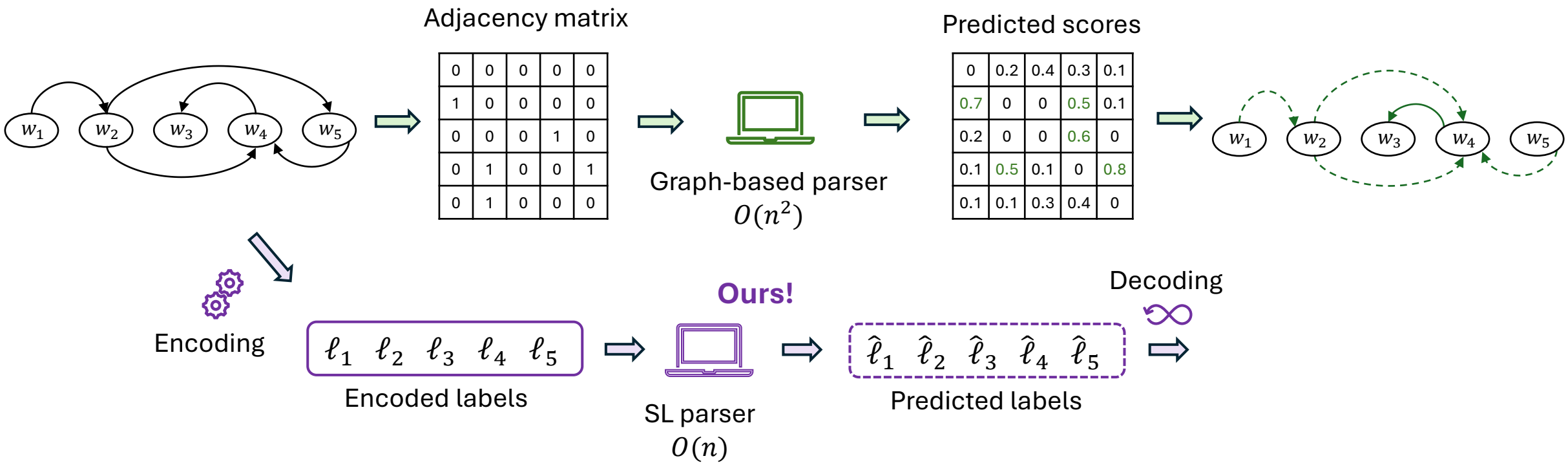

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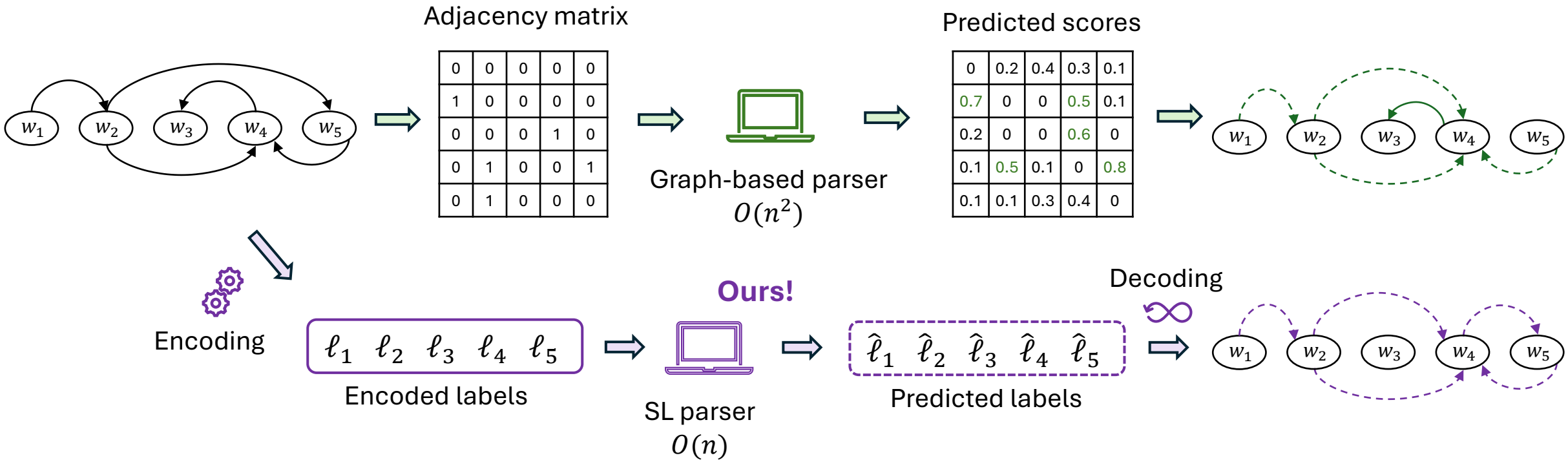

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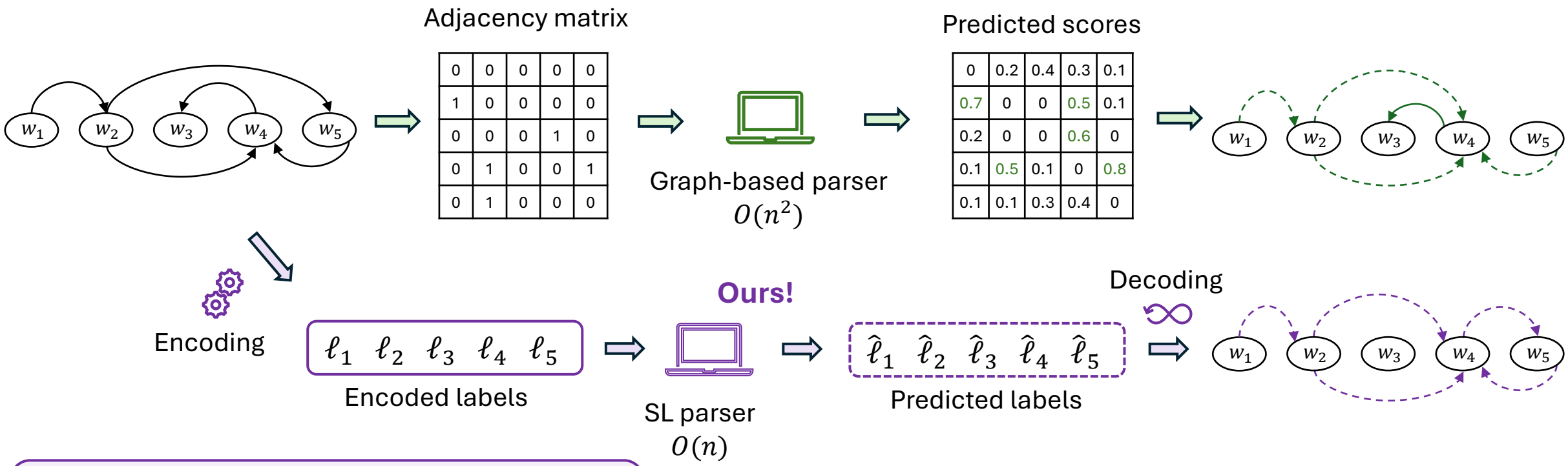


1. Define the encoding and decoding functions.
2. Then tackle as a classical tagging task

... as **Sequence Labeling**.

We propose various encoding ( ) and decoding ( ) functions

Unbounded Encodings

Positional Encodings

Absolute
Relative

k-planar Bracketing Encoding

</\>

Non-fixed number of unique labels.

Bounded Encodings

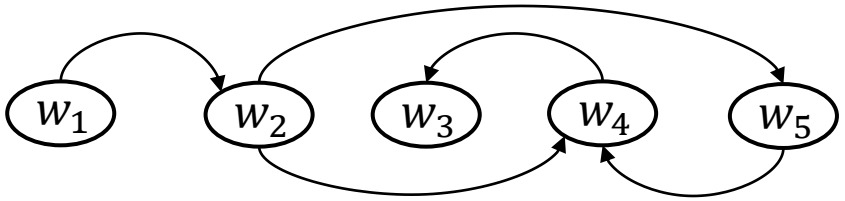
$4k$-bit Encoding

$6k$-bit Encoding

Fixed number of unique labels.

Positional encodings: absolute (A) and relative (R)

- **Absolute**: Sorted sequence of head positions.

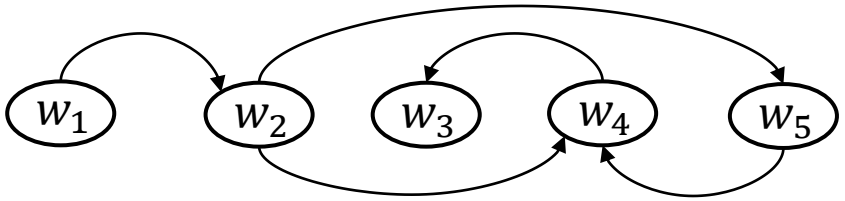


	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5
Absolute:	(,)	(1)	(4)	(2,5)	(2)

Adapted from Strzyz et al. (2019)

Positional encodings: absolute (A) and relative (R)

- **Absolute**: Sorted sequence of head positions.
- **Relative**: Sorted sequence of head relative positions.



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Absolute:	(,)	(1)	(4)	(2,5)	(2)
Relative:	(,)	(-1)	(1)	(-2, 1)	(-3)

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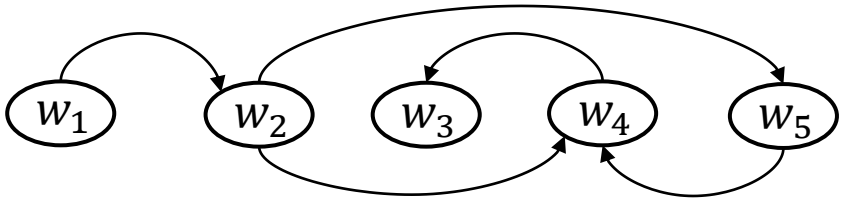
Positional encodings: absolute (A) and relative (R)

- **Absolute**: Sorted sequence of head positions.
- **Relative**: Sorted sequence of head relative positions.

Fast and simple decoding (easy to parallelize).



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k-planar Bracketing Encoding

$\ell_i \sim$ < / \ >

Adapted from Strzyz et al. (2019)

k-planar Bracketing Encoding

$\ell_i \sim$ <

/

\

>

$\ell_i =$ <</

Adapted from Strzyz et al. (2019)

k-planar Bracketing Encoding

$\ell_i \sim$

< : There is a head at the right.

/

\

>

Adapted from Strzyz et al. (2019)

k-planar Bracketing Encoding

$\ell_i \sim$

- `<` : There is a head at the right.
- `/` : There is a dependent at the right.
- `\`
- `>`

Adapted from Strzyz et al. (2019)

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- `<` : There is a head at the right.
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- Distribute the arcs in planes (non-crossing arcs).
- Add symbol `*` for brackets of other planes.
- First plane: `</\>`.
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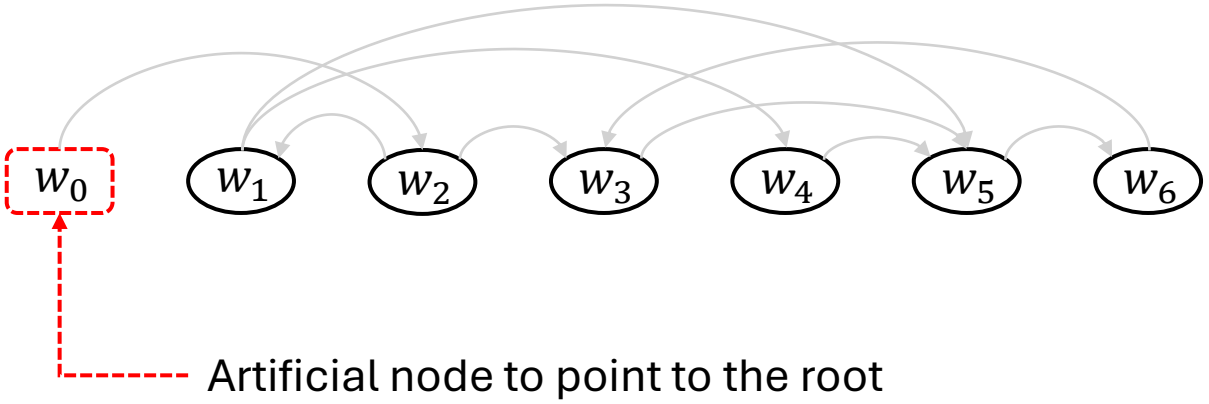

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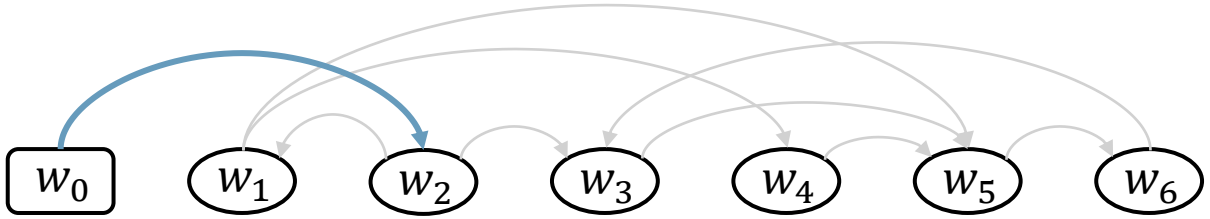

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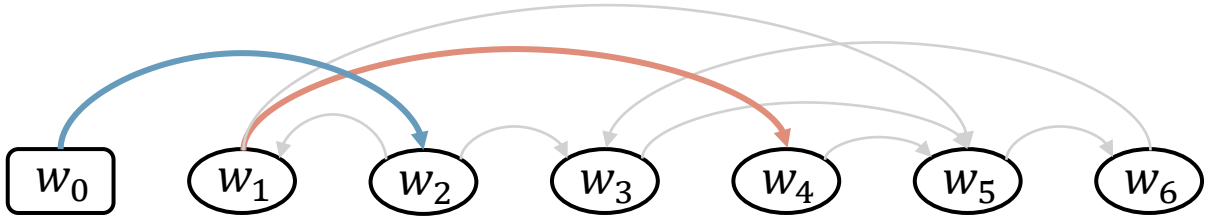

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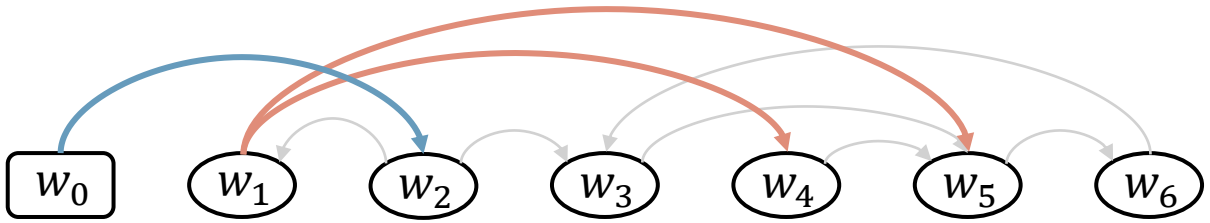

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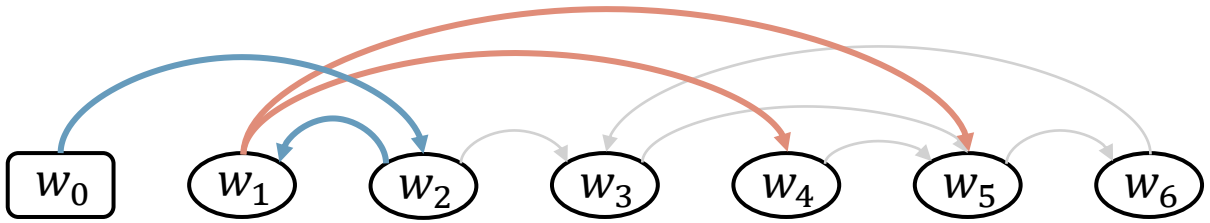

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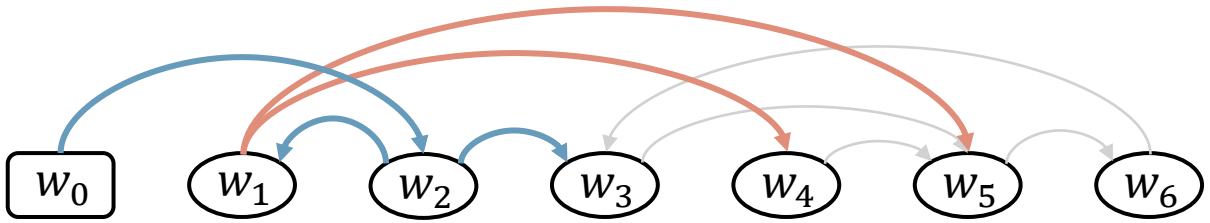

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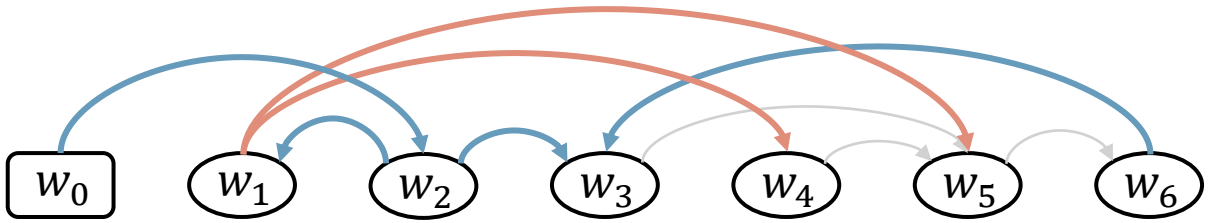

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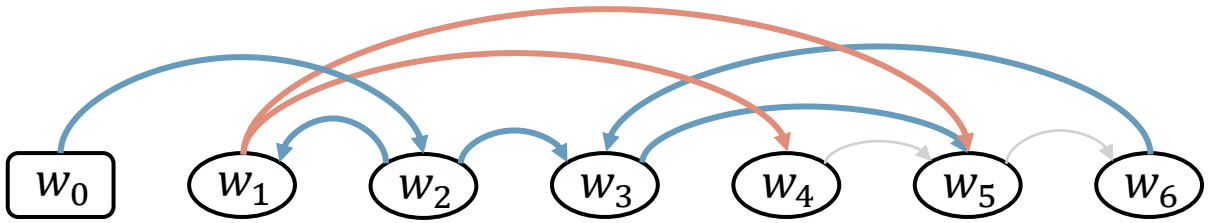

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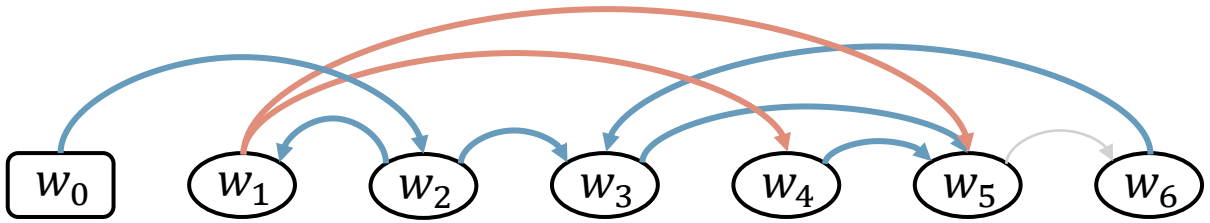


Adapted from Strzyz et al. (2019)

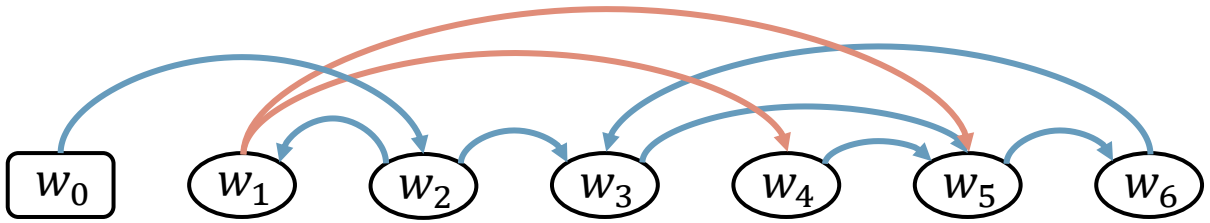
k-planar Bracketing Encoding

$\ell_i \sim$

- `<` : There is a head at the right.
- `/` : There is a dependent at the right.
- `\` : There is a dependent at the left.
- `>` : There is a head is at the left.

- Distribute the arcs in planes (non-crossing arcs).
- Add symbol `*` for brackets of other planes.
- First plane: `</\>`.
- Second plane: `<*/*\*>*`.
- Third plane: `<**/**\**>**`.

Step 1: Distribute in planes.



Adapted from Strzyz et al. (2019)

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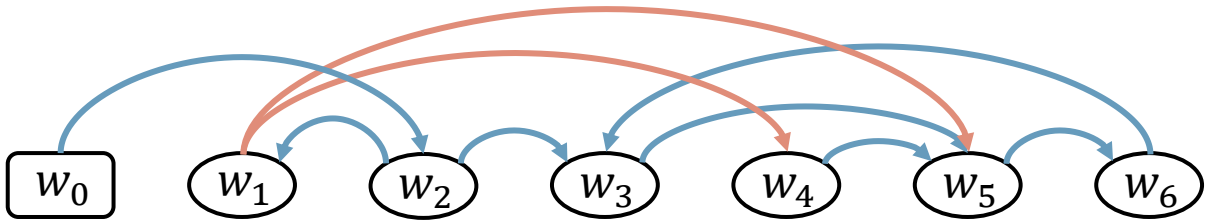
- Add symbol * for brackets of other planes.
- First plane: </\>.
- Second plane: <*/**>*.
- Third plane: <**/****>**.

Step 2: Assign labels to each plane.



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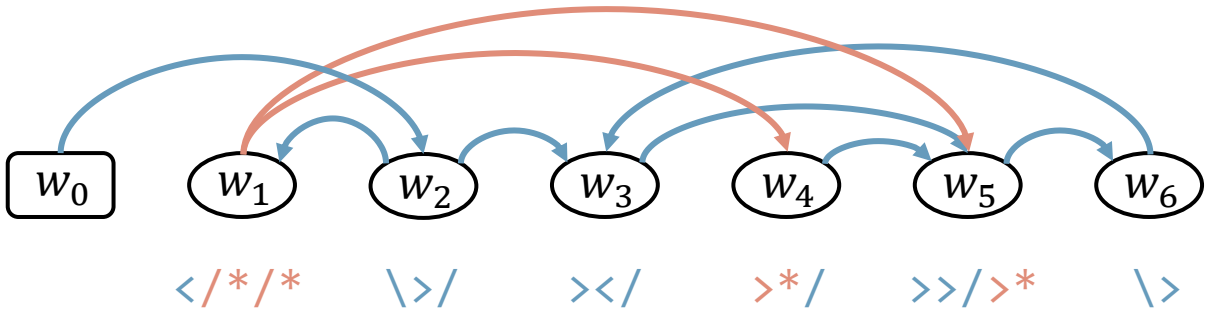

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Step 2: Assign labels to each plane.

w_0	w_1	w_2	w_3	w_4	w_5	w_6
	`</*/*`	`\>/`	`></`	`>*/`	`>>/>*`	`\>`

We predict this!

Adapted from Strzyz et al. (2019)

k-planar Bracketing Encoding

How do we recover the arcs from the labels $(\ell_1, \dots, \ell_n)$?

Adapted from Strzyz et al. (2019)

k-planar Bracketing Encoding

- Two stacks (**L** and **R**) per plane.
- **L** for left arcs (**<**)and **R** right arcs (**/>**).

Adapted from Strzyz et al. (2019)

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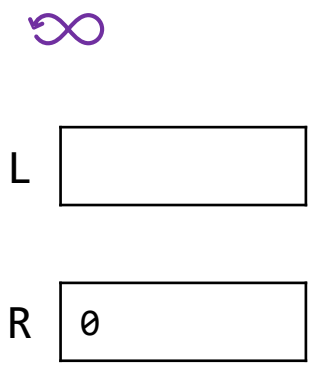

Adapted from Strzyz et al. (2019)

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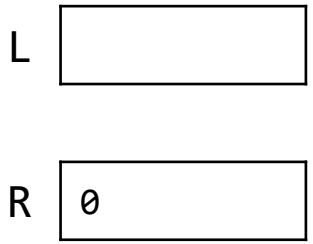

Adapted from Strzyz et al. (2019)

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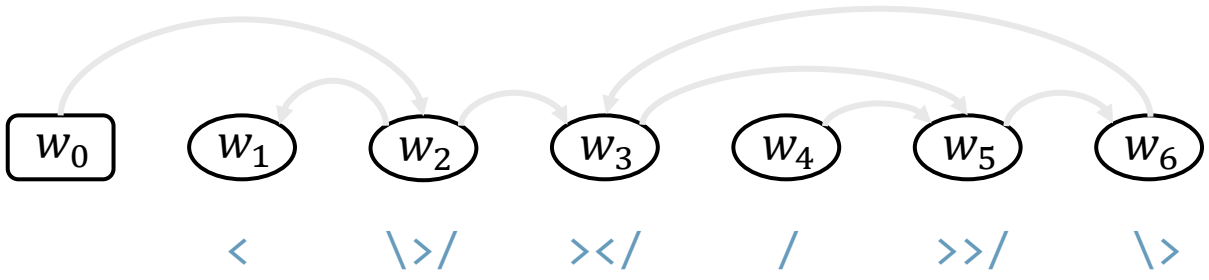
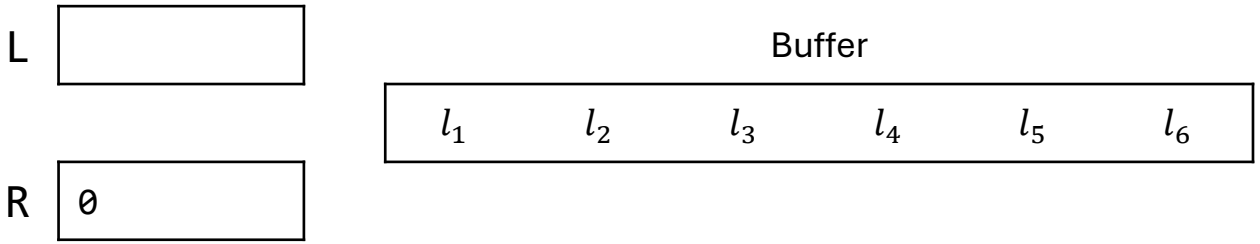


Initial state: Labels to buffer and **R** with the root dependency.



Adapted from Strzyz et al. (2019)

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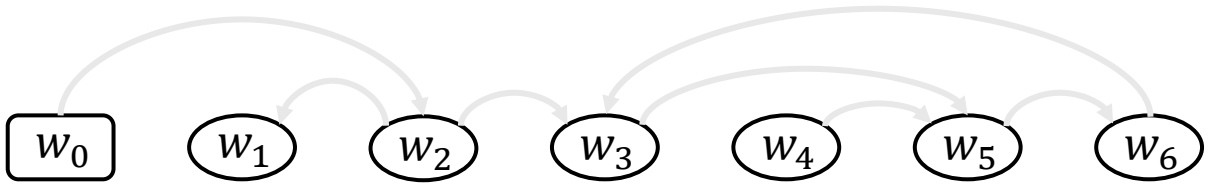

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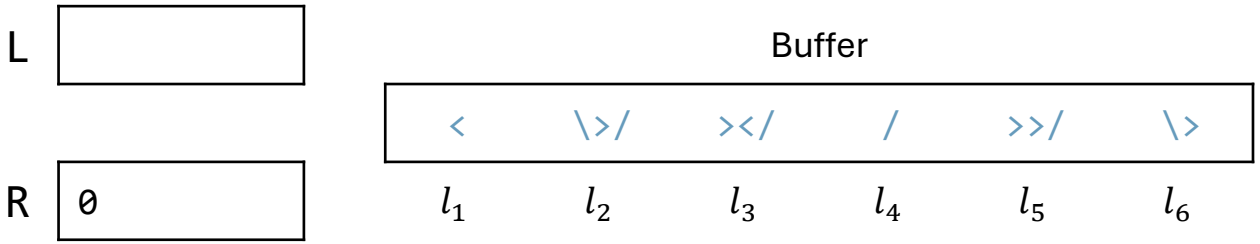

Adapted from Strzyz et al. (2019)

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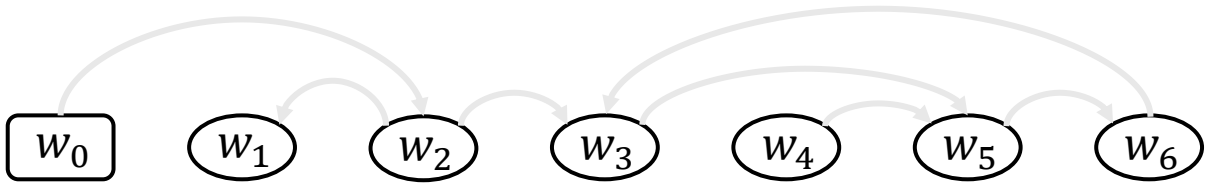

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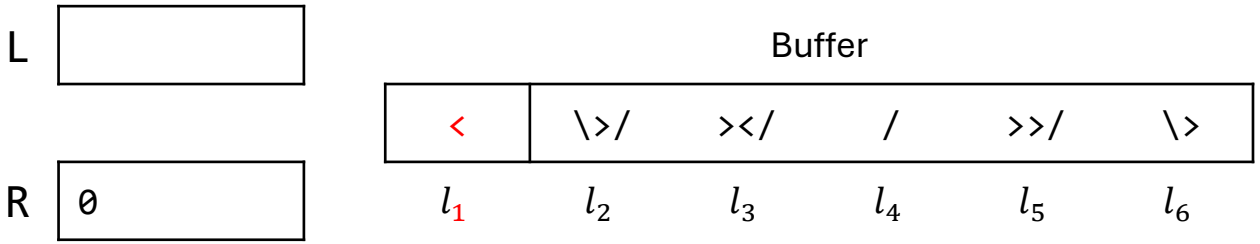

Adapted from Strzyz et al. (2019)

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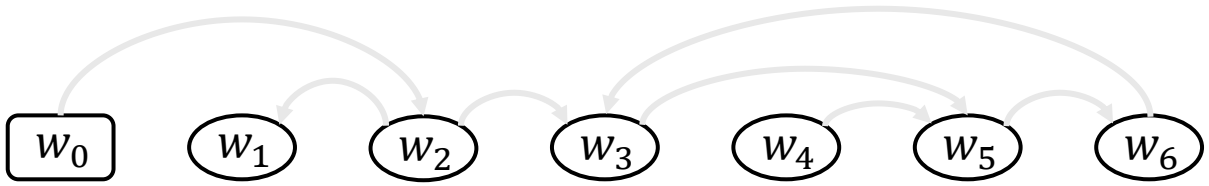

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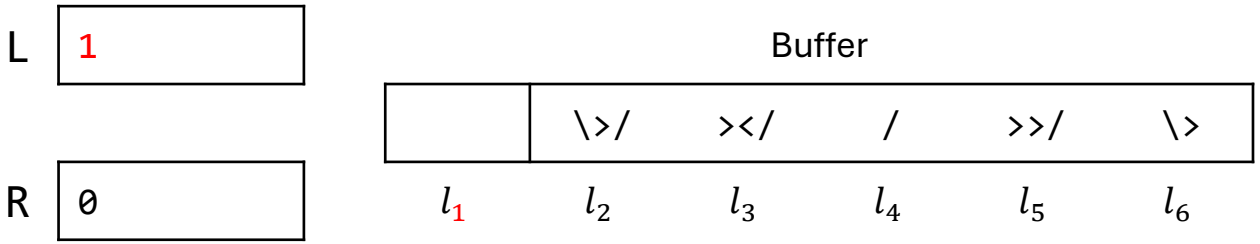

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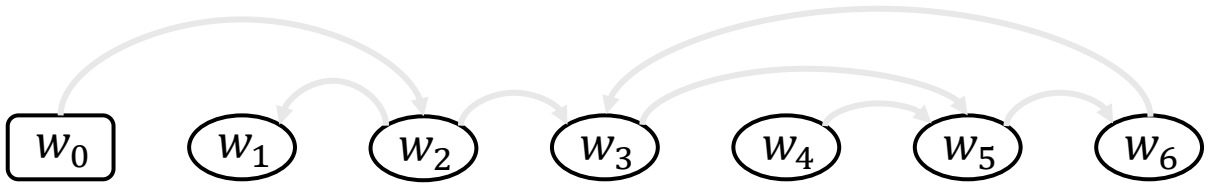

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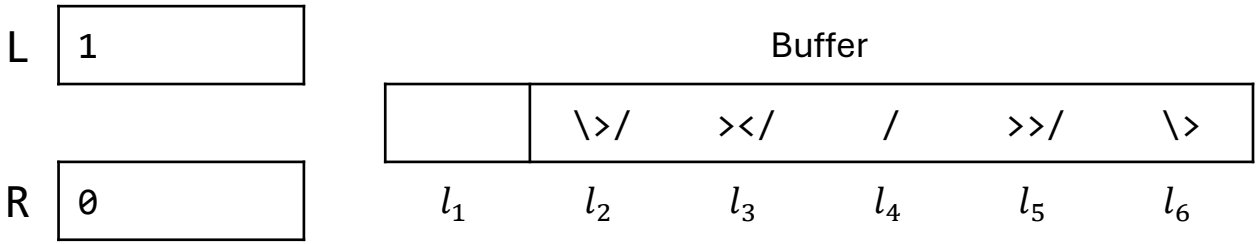

Adapted from Strzyz et al. (2019)

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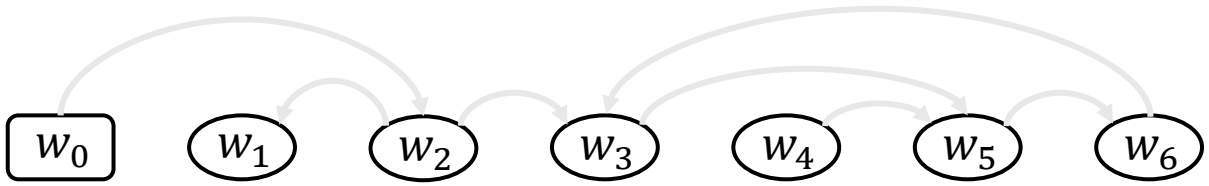

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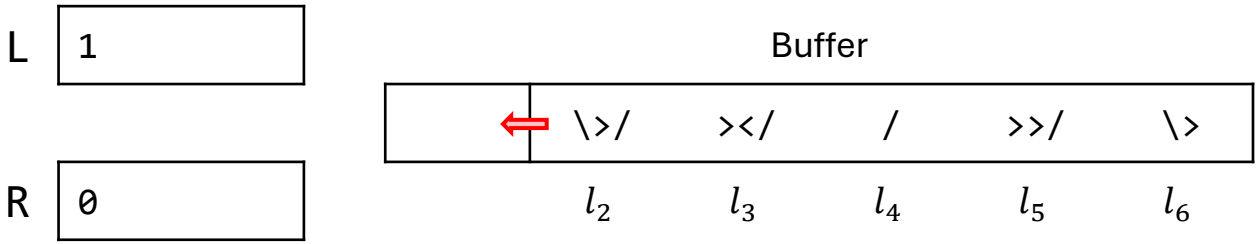

Adapted from Strzyz et al. (2019)

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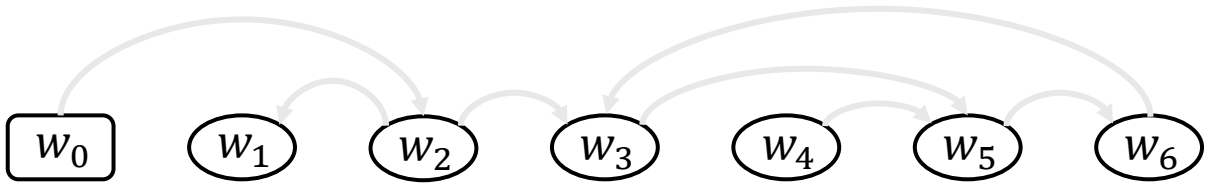
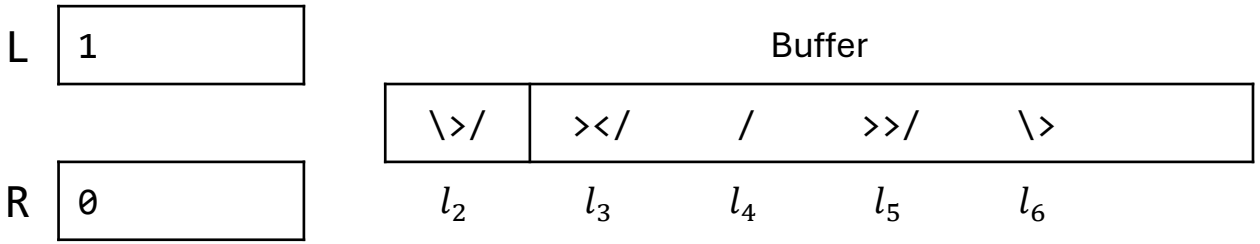


Step 2.



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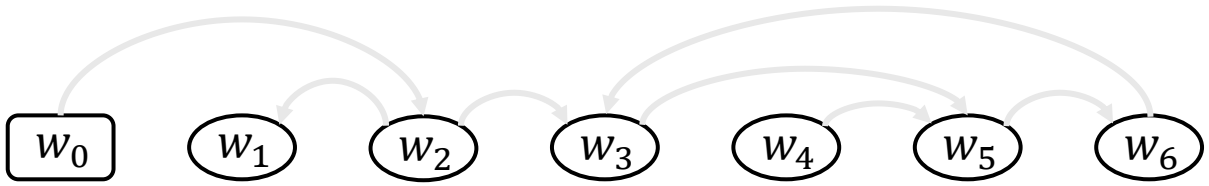

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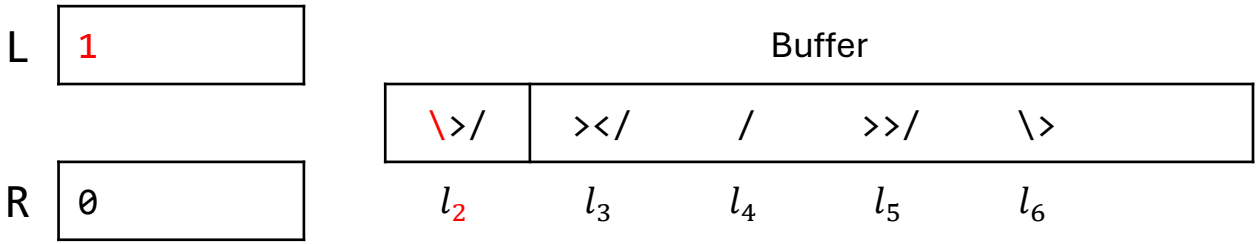

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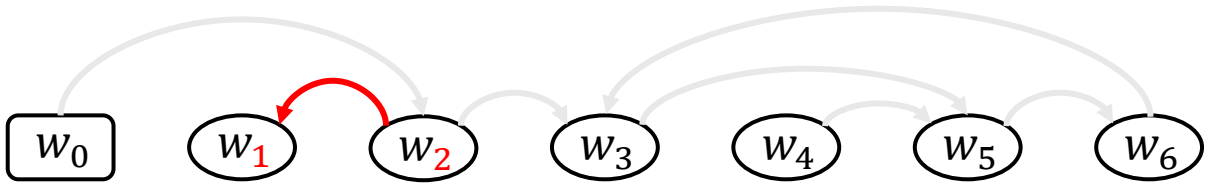

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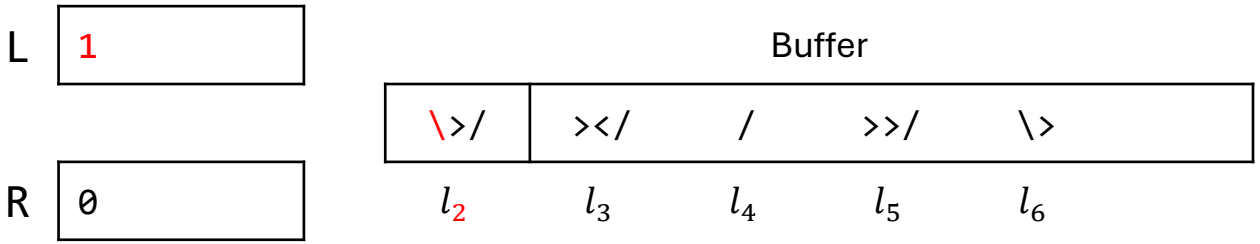

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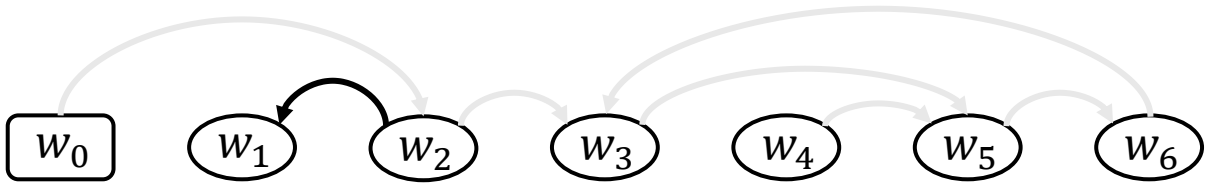

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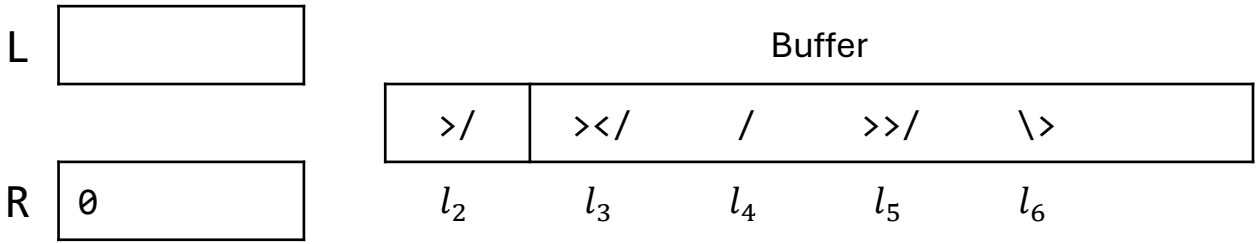

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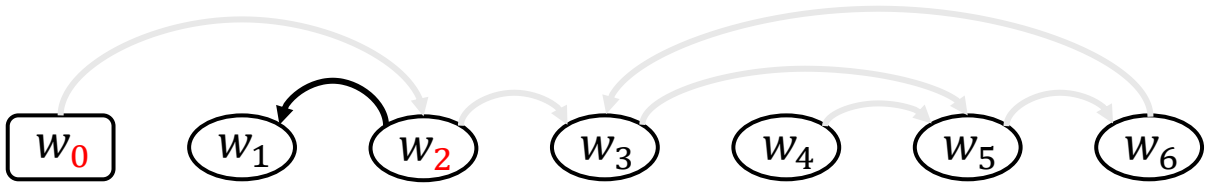

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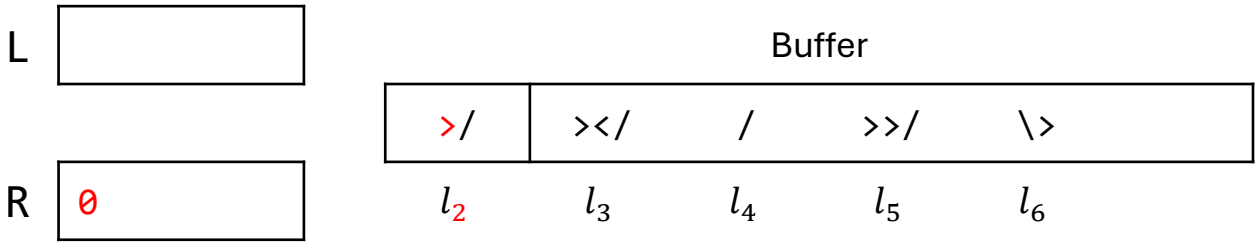

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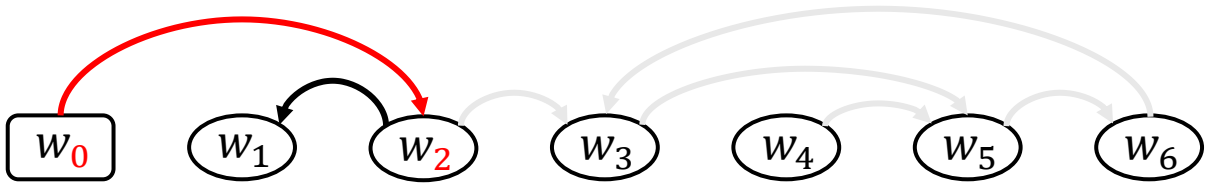

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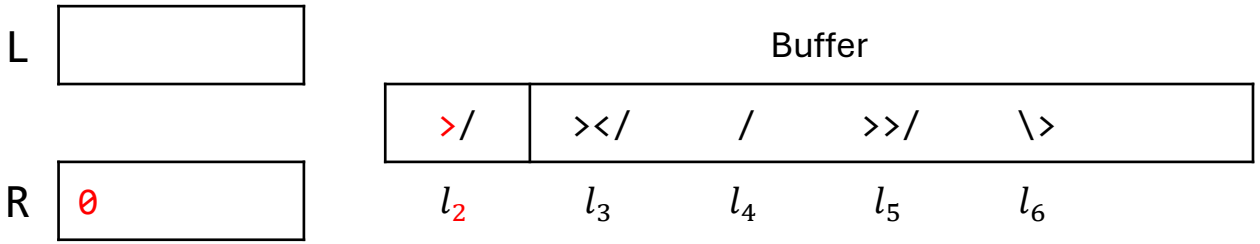

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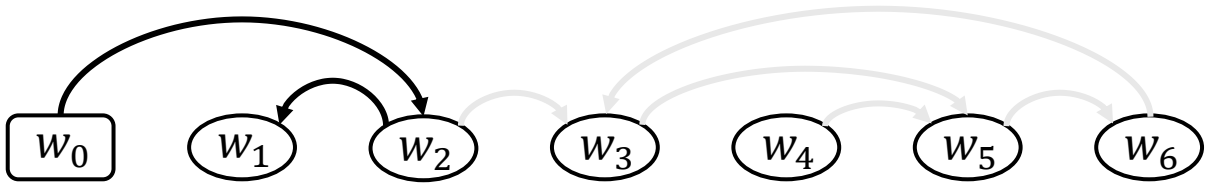

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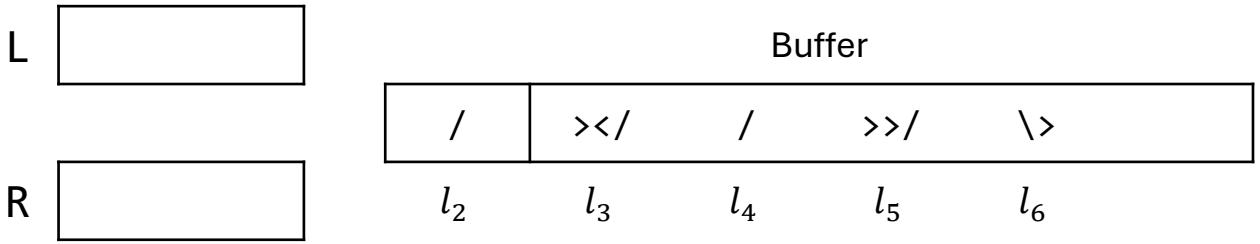

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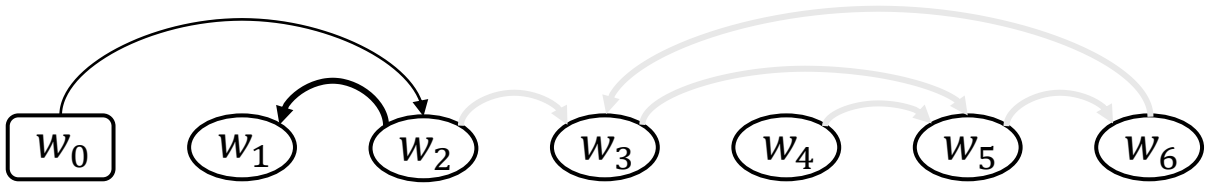

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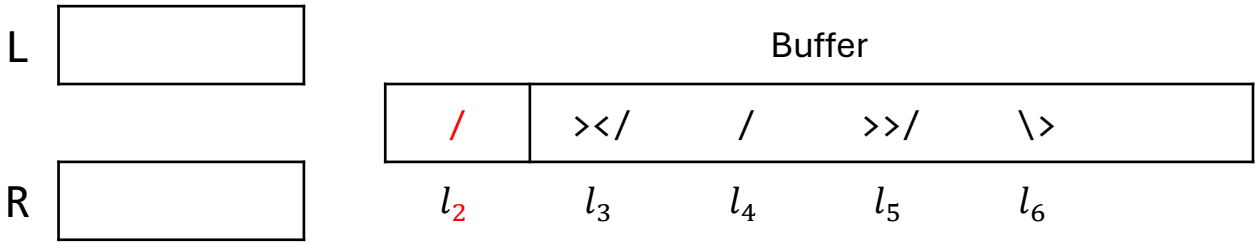

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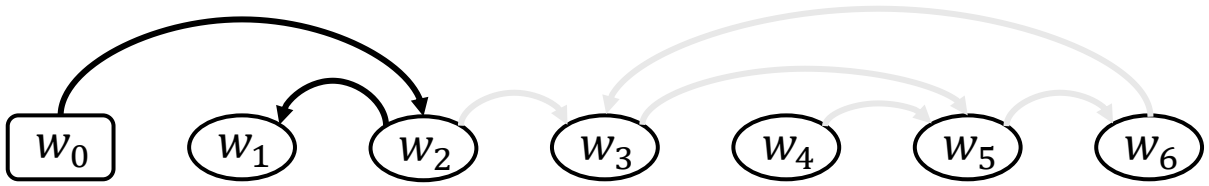
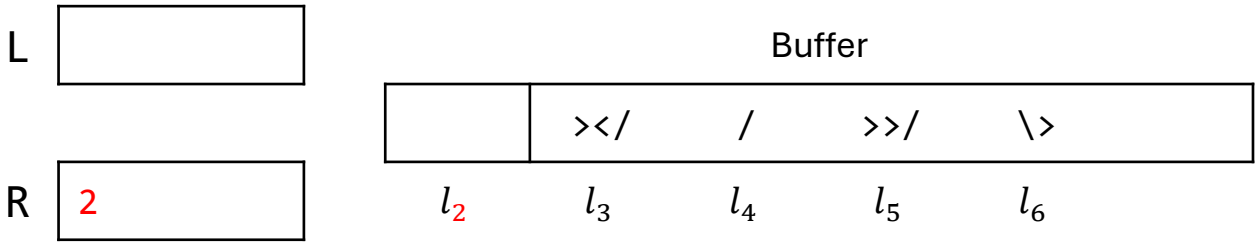


Step 2.



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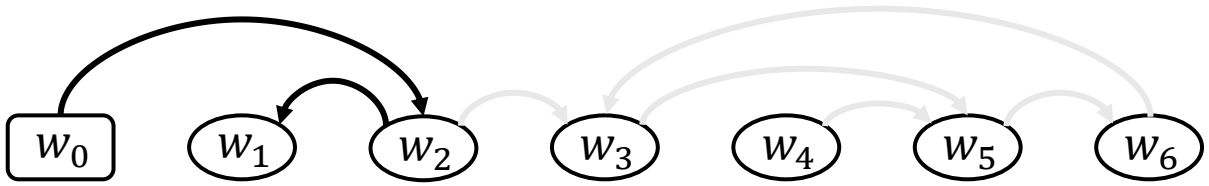

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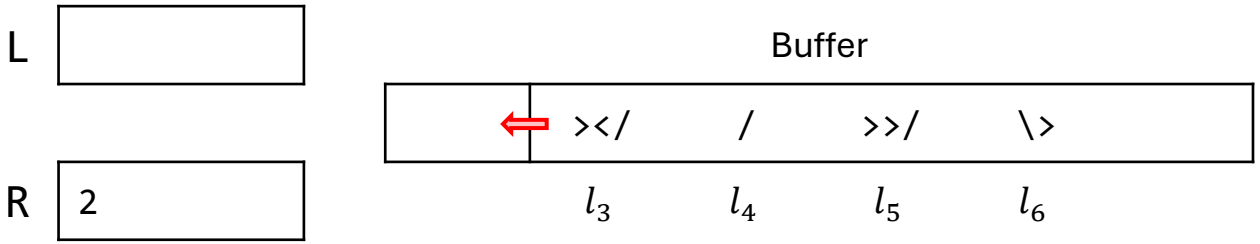

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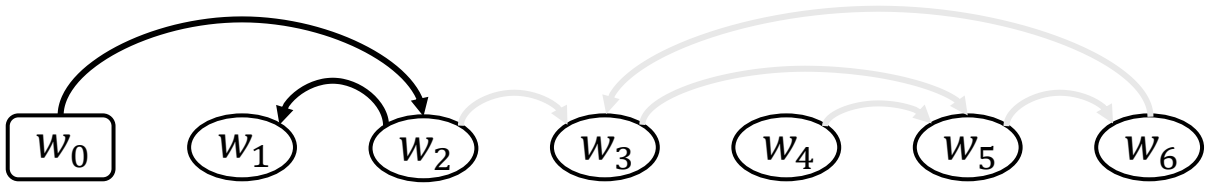
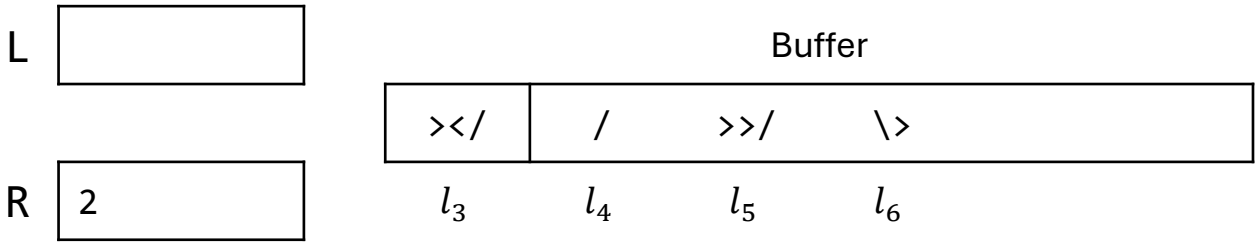


Step 3.



Adapted from Strzyz et al. (2019)

k-planar Bracketing Encoding

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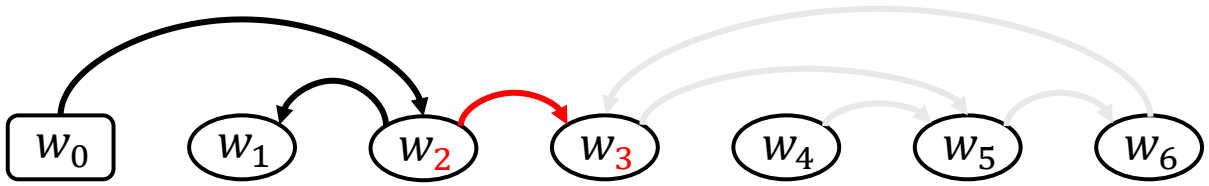

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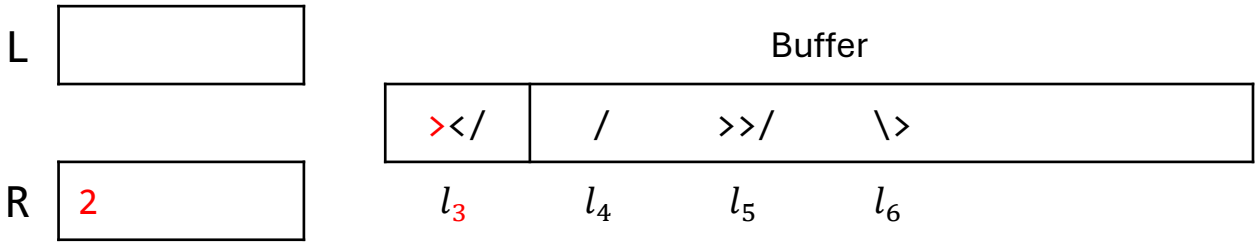

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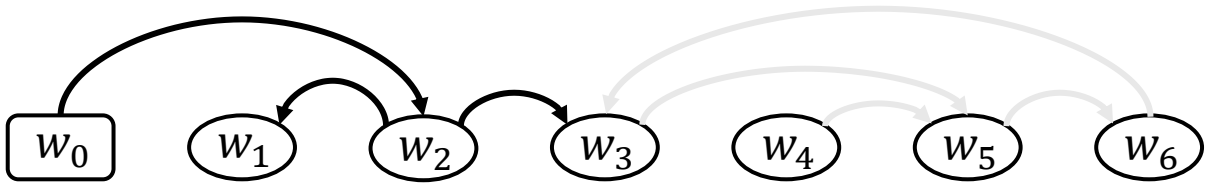

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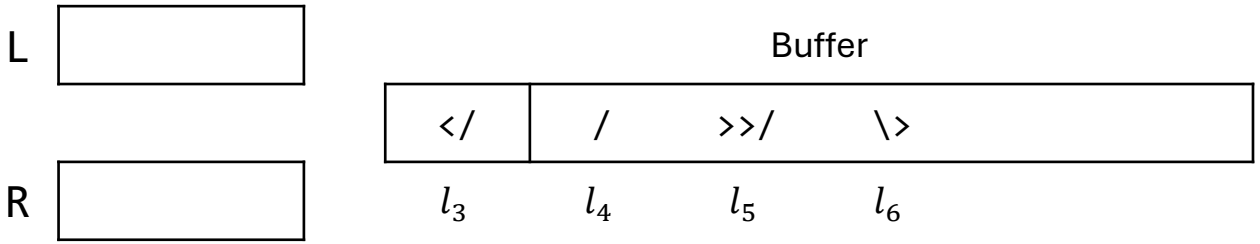

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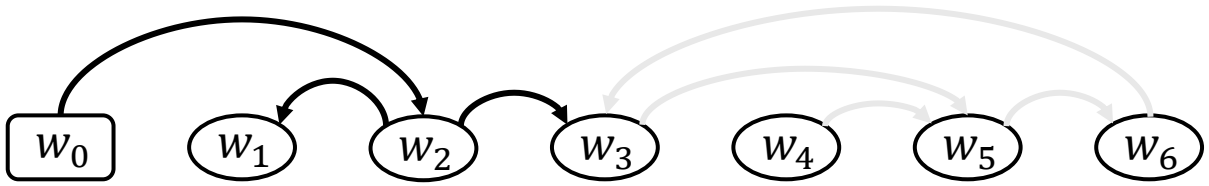

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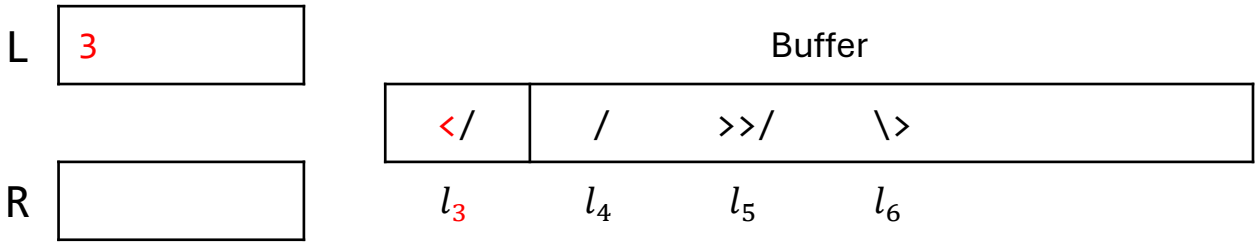

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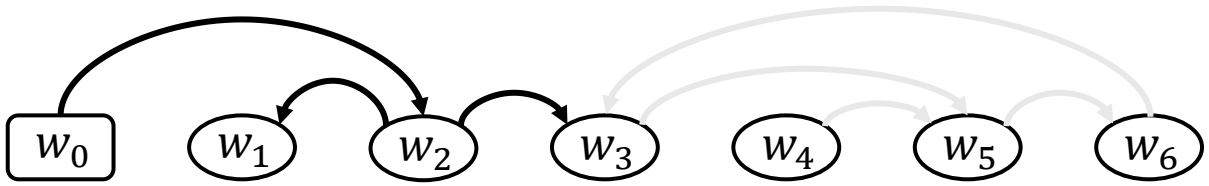

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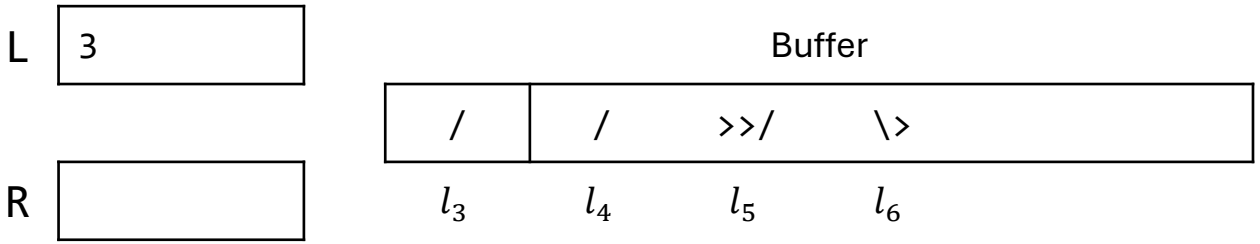

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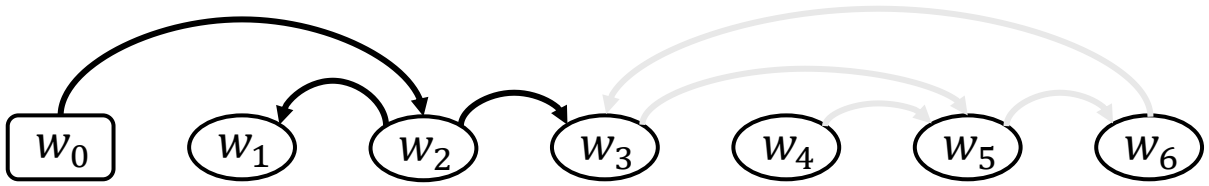

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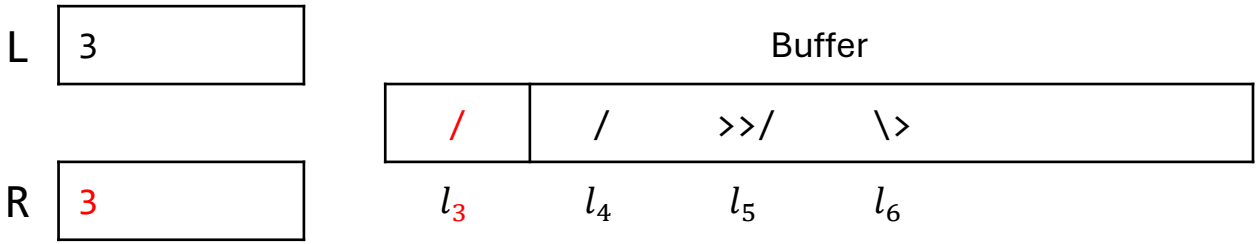

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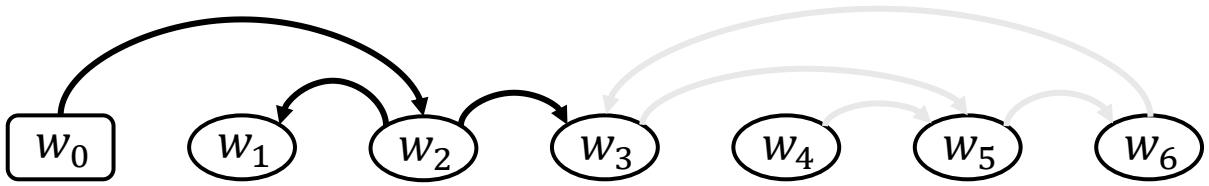

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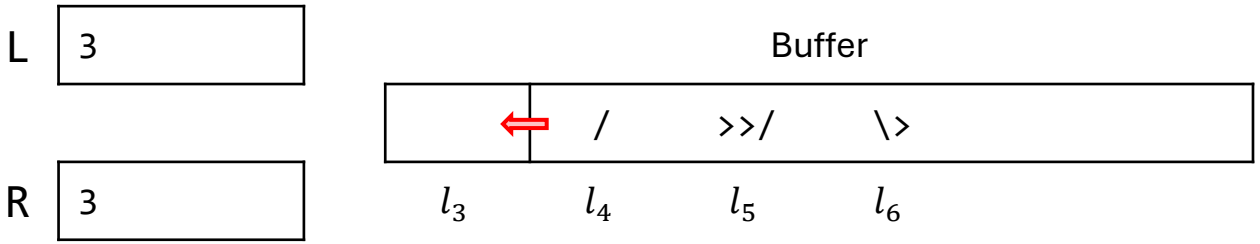

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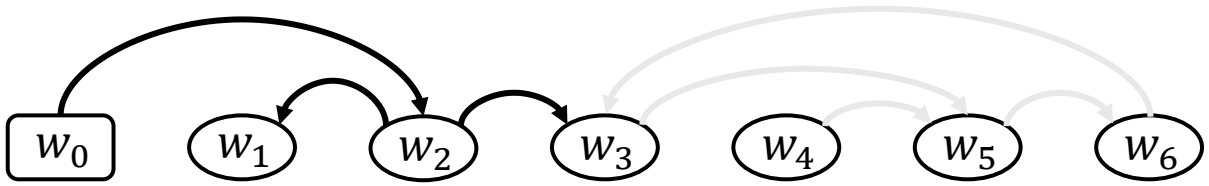
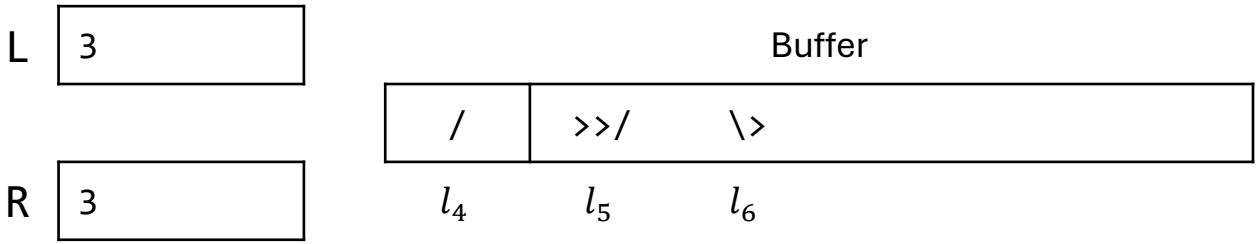


Step 4.



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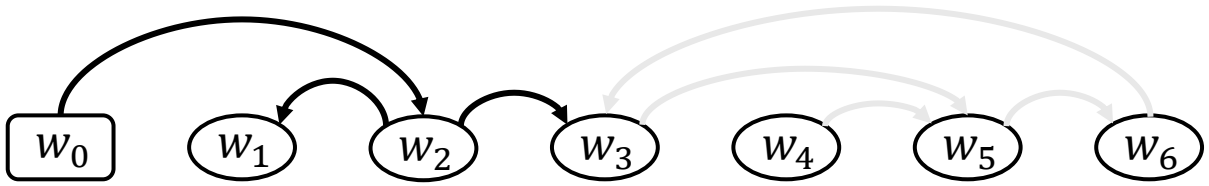

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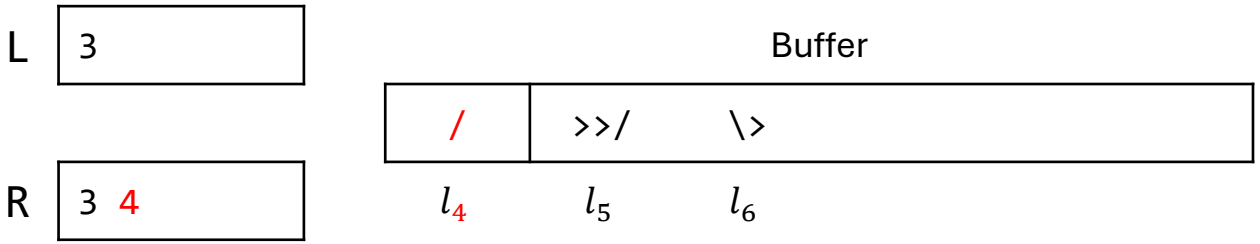

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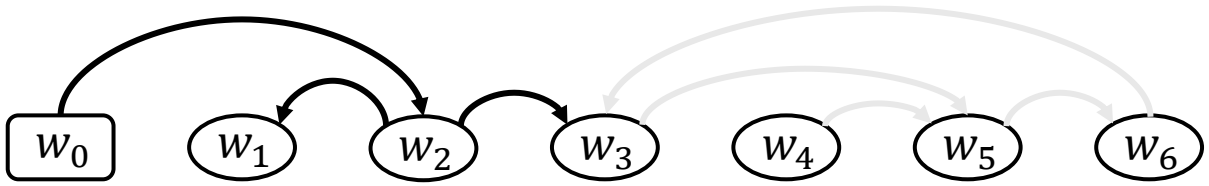

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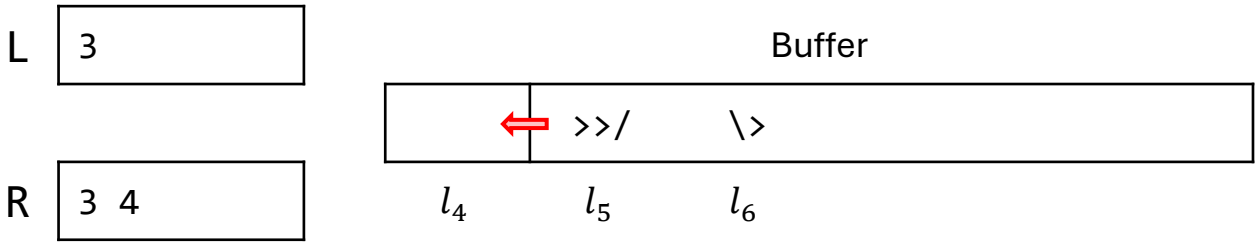

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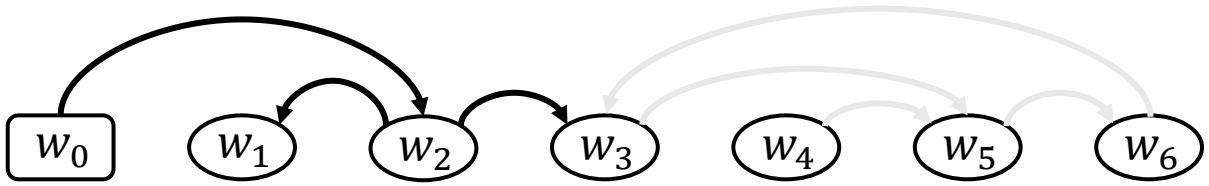

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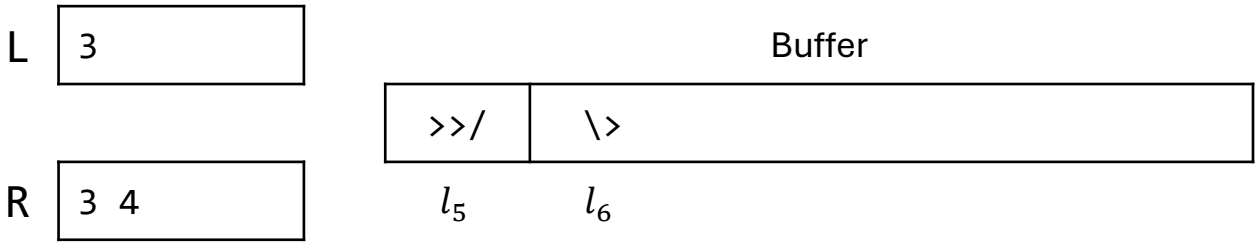

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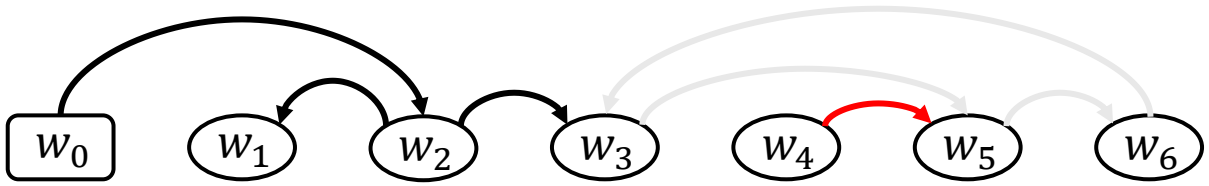

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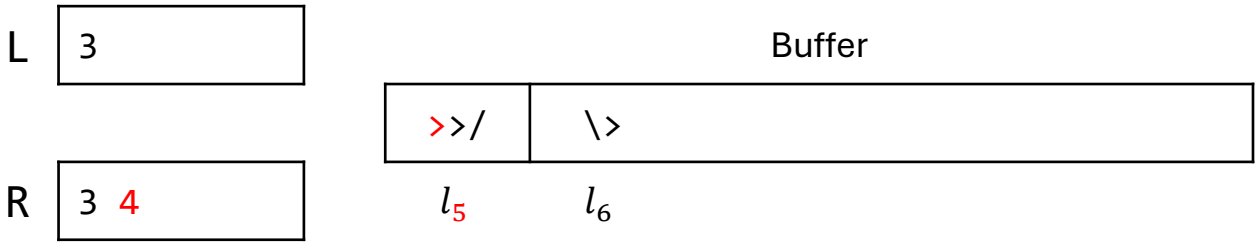

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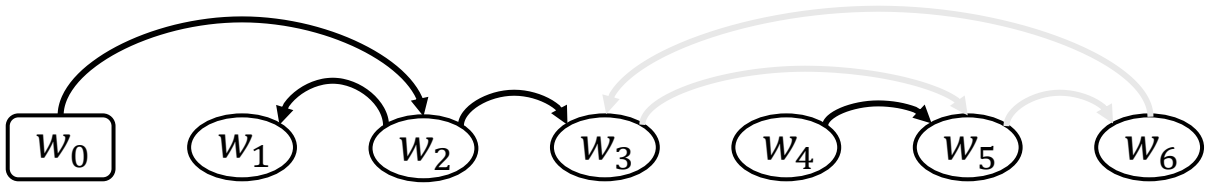

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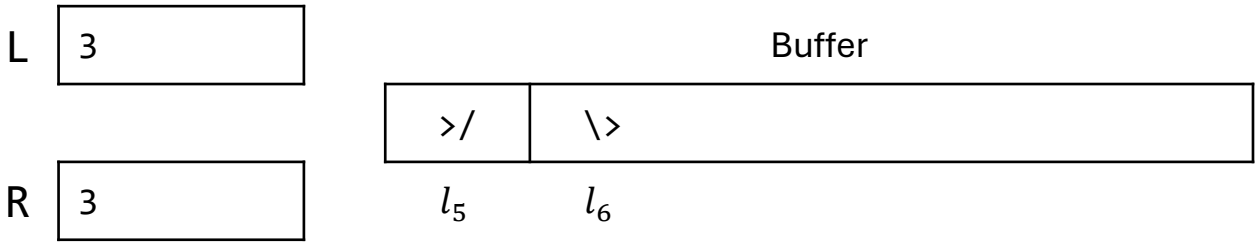

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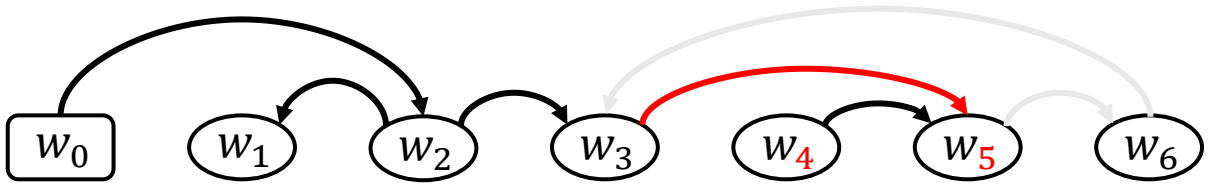

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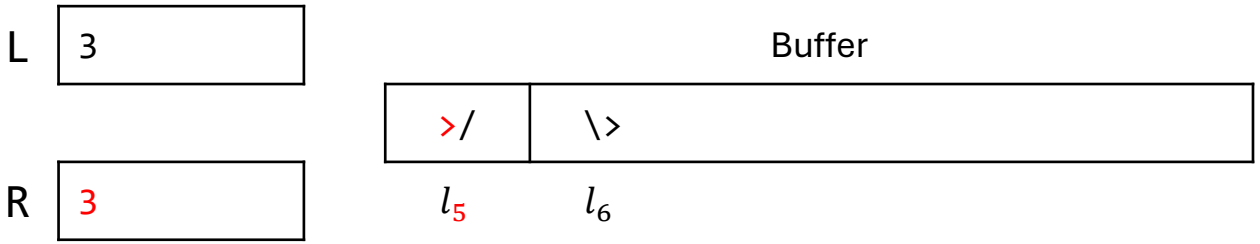

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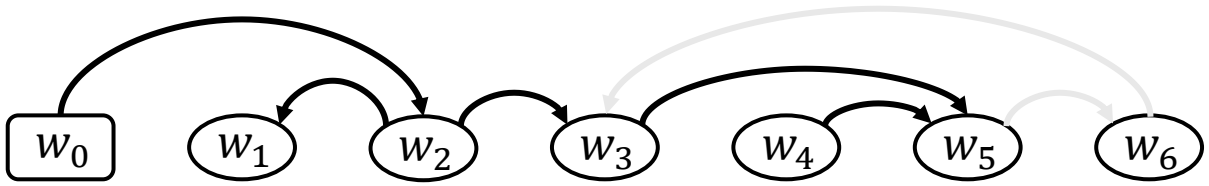

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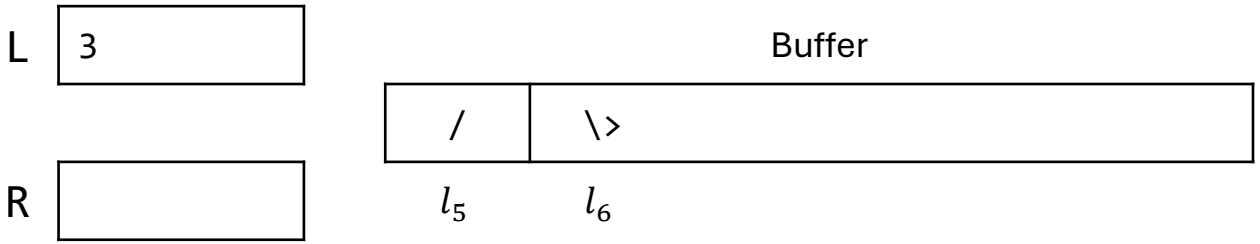

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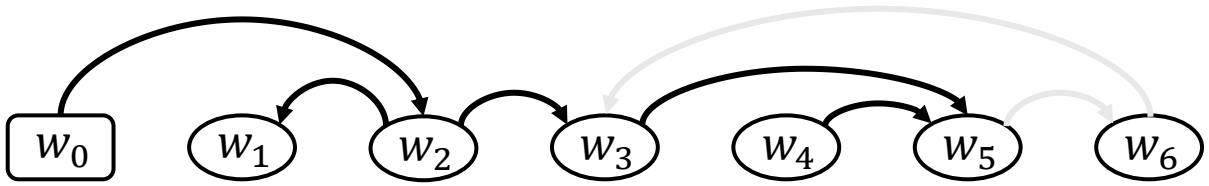

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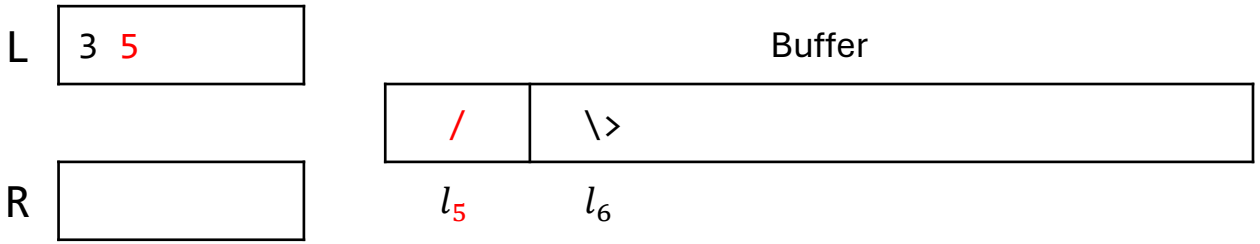

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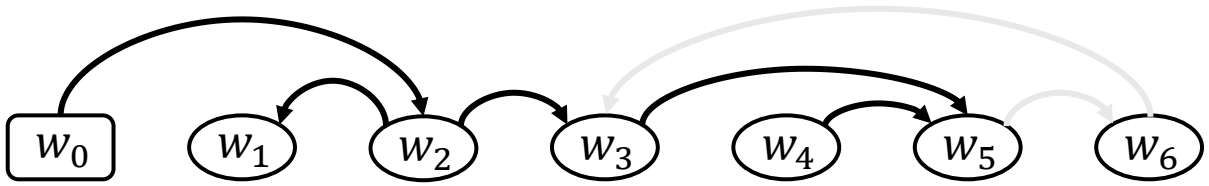

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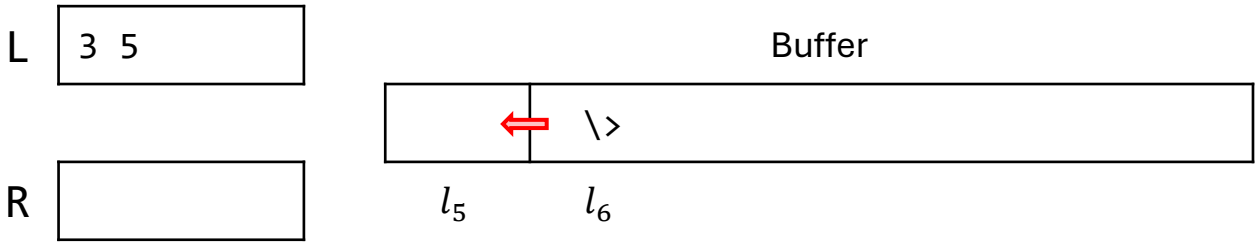

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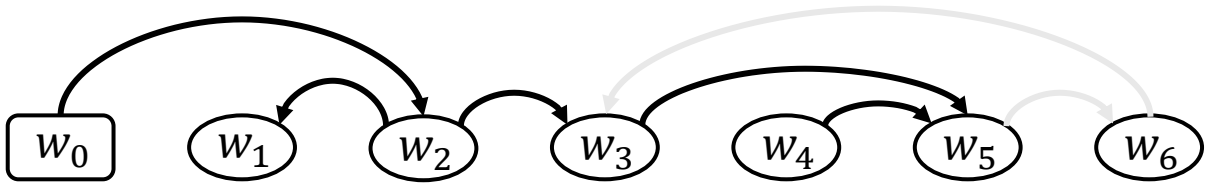

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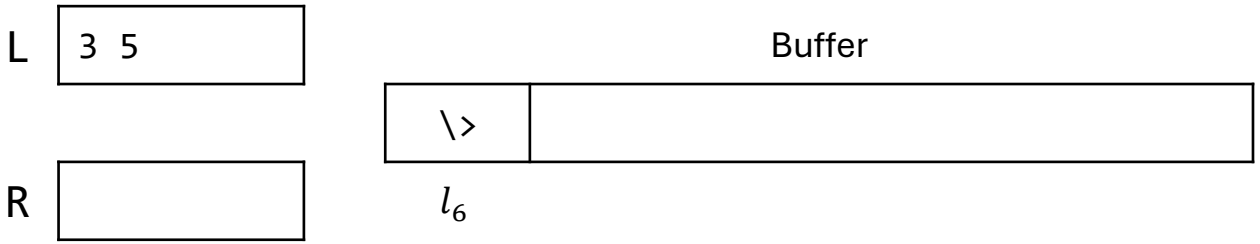

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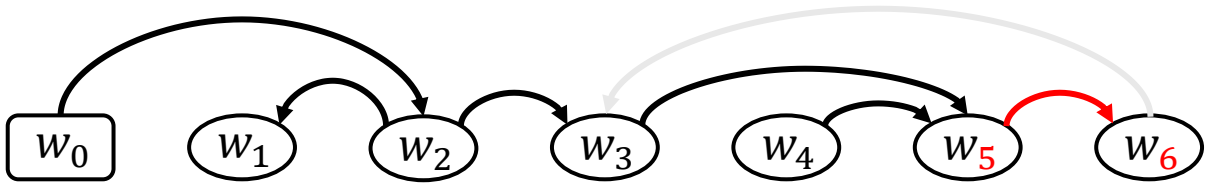

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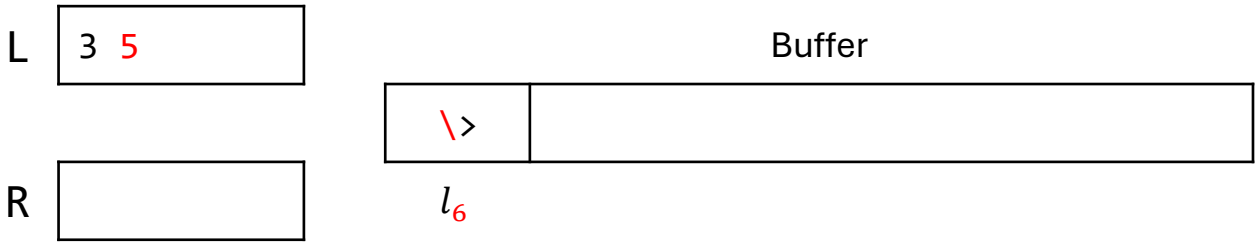

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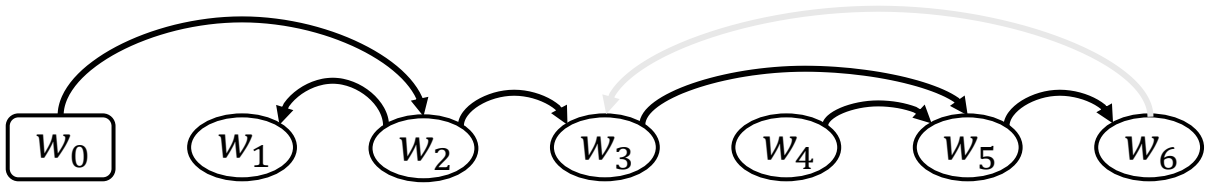

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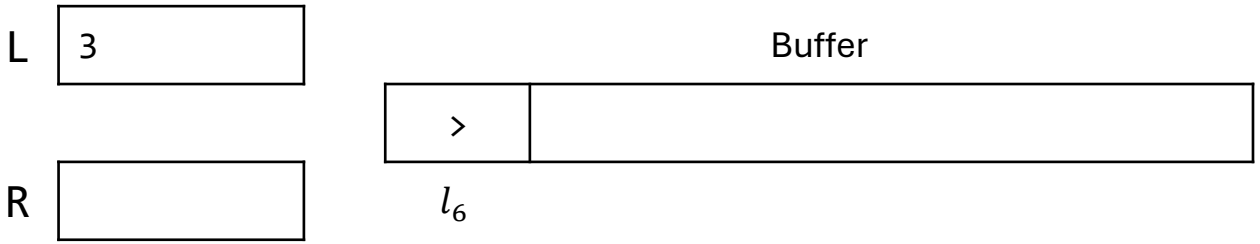

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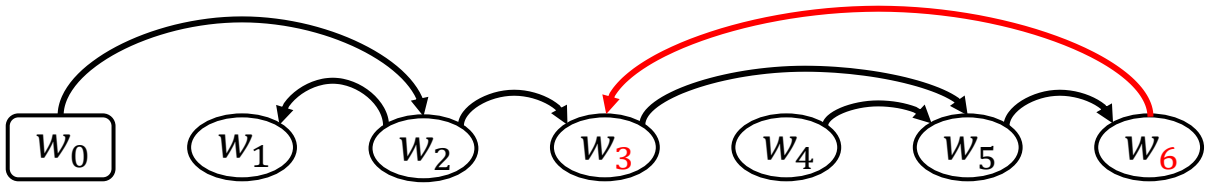

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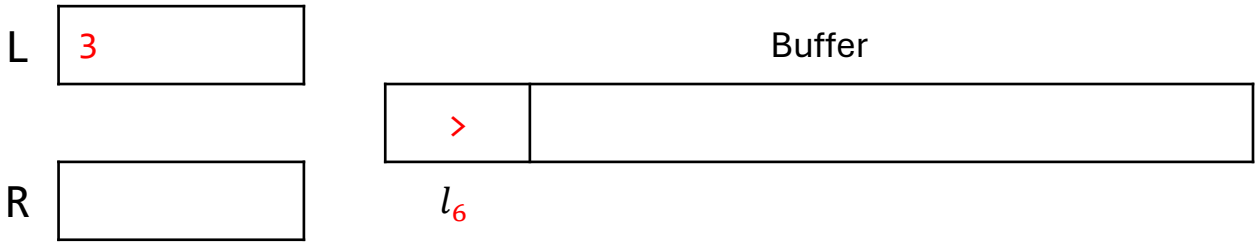

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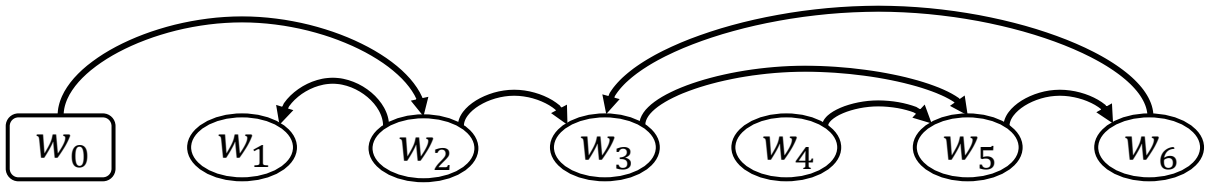
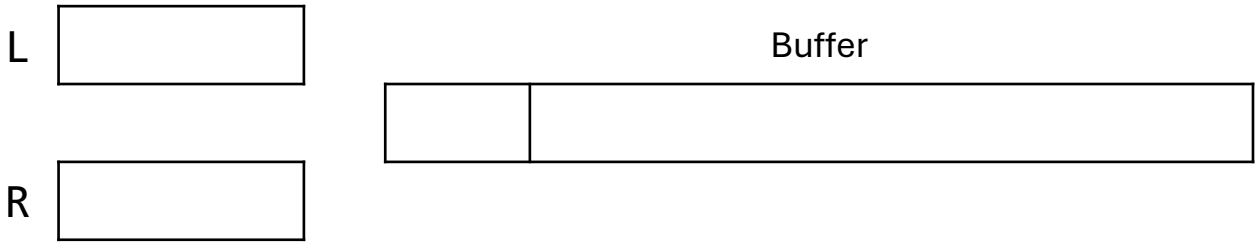


Finished!



Adapted from Strzyz et al. (2019)

Some terminology...

1-planar graph

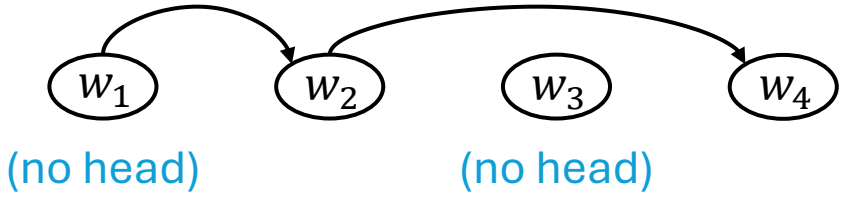
- No crossing arcs.

Relaxed 1-planar *graph*

- No crossing arcs *in the same direction*.

Relaxed 1-planar *tree*

- No crossing arcs *in the same direction*.
- *Only one* head per node.



✓ 1-planar

✓ Relaxed 1-planar graph

✗ Relaxed 1-planar tree

Some terminology...

1-planar graph

- No crossing arcs.

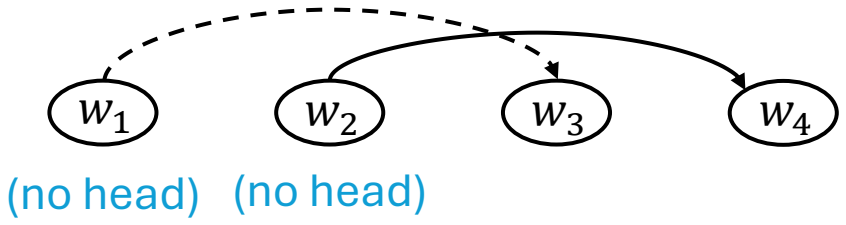
Relaxed 1-planar *graph*

- No crossing arcs *in the same direction*.

Relaxed 1-planar *tree*

- No crossing arcs *in the same direction*.
- *Only one* head per node.

(crossing arcs in the same direction)



✗ 1-planar

✗ Relaxed 1-planar graph

✗ Relaxed 1-planar tree

Some terminology...

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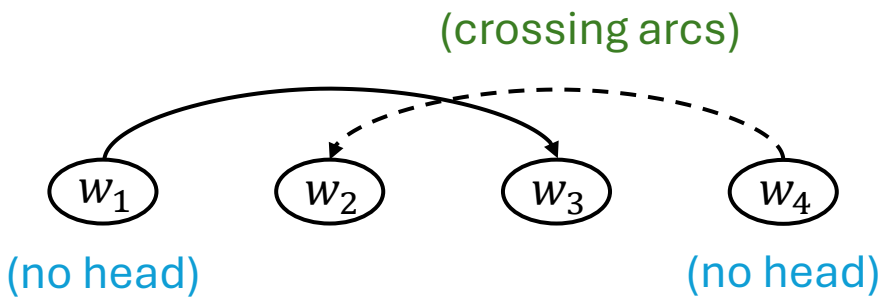

✗ 1-planar

✓ Relaxed 1-planar graph

✗ Relaxed 1-planar tree

Some terminology...

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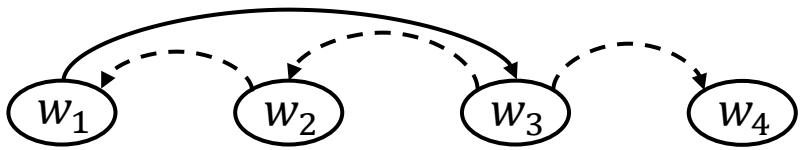

✓ 1-planar

✓ Relaxed 1-planar graph

✓ Relaxed 1-planar tree

$4k$-bit Encoding

Assumptions:

- Distribute the arcs in **relaxed 1-planar trees.**
- Encode each label ℓ_i with 4 bits: $\mathbf{b_0 b_1 b_2 b_3}$.

$\mathbf{b_0}$	**0**	w_i has a right head.
	1	w_i has a left head.
$\mathbf{b_1}$		w_i is the outermost dependent.
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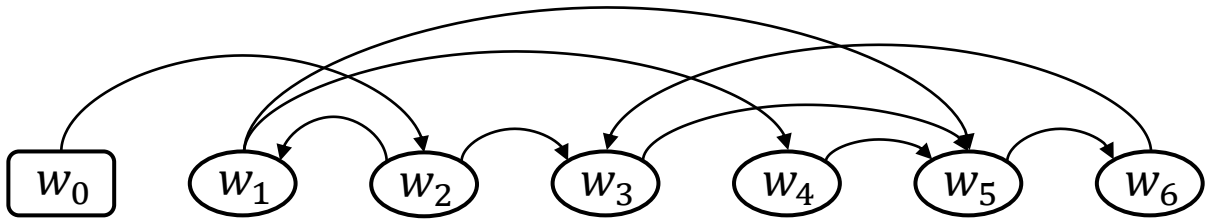

Sort and incrementally assign!

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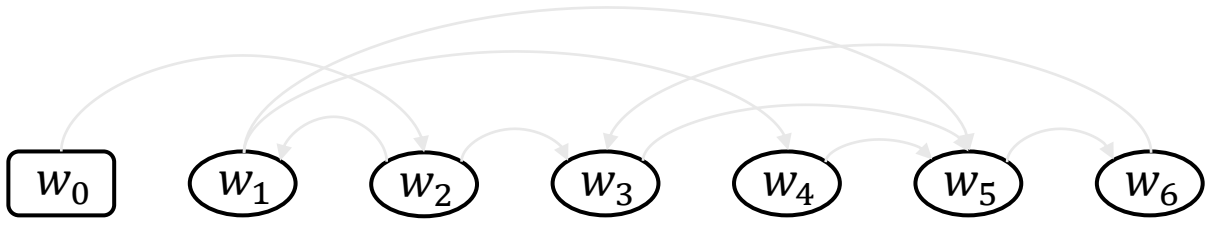

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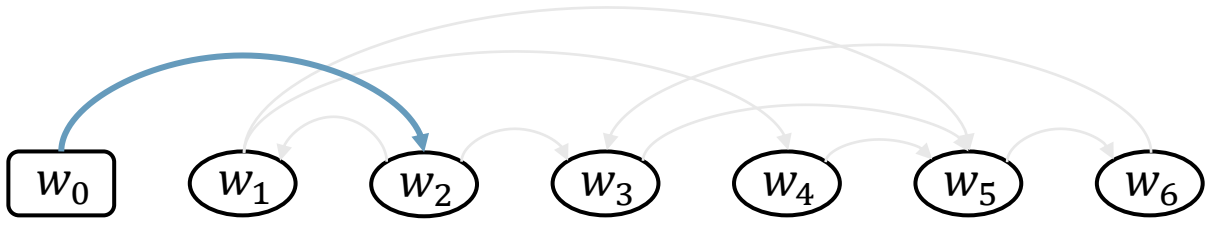

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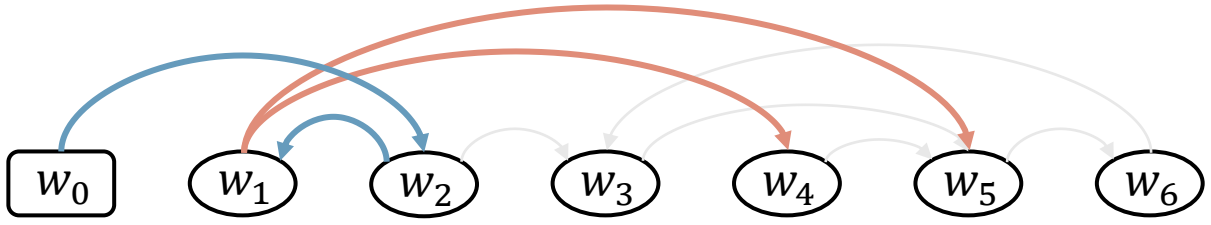

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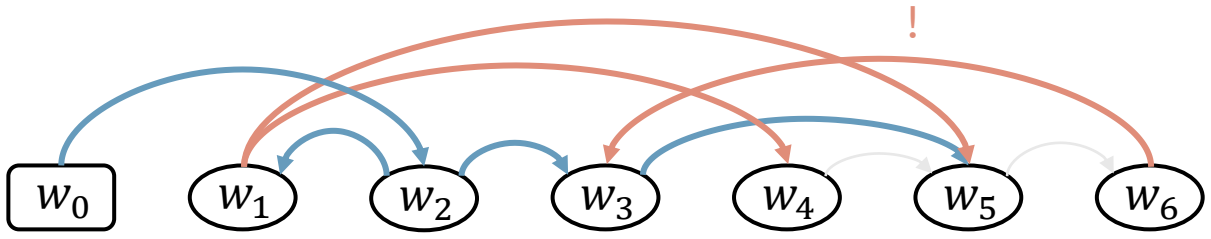

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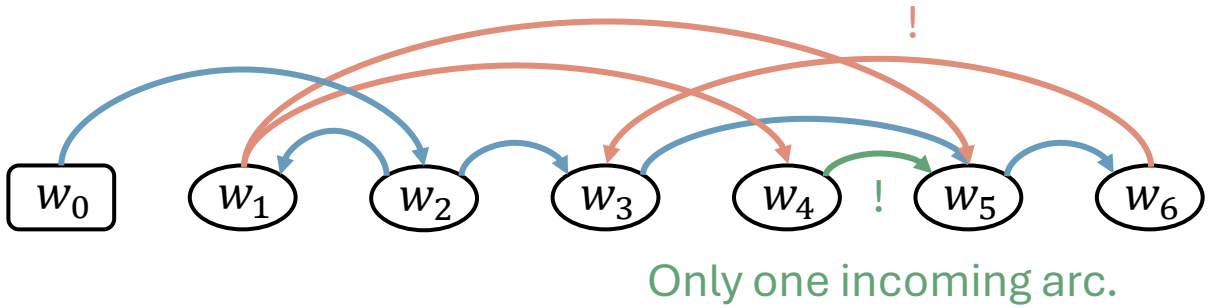

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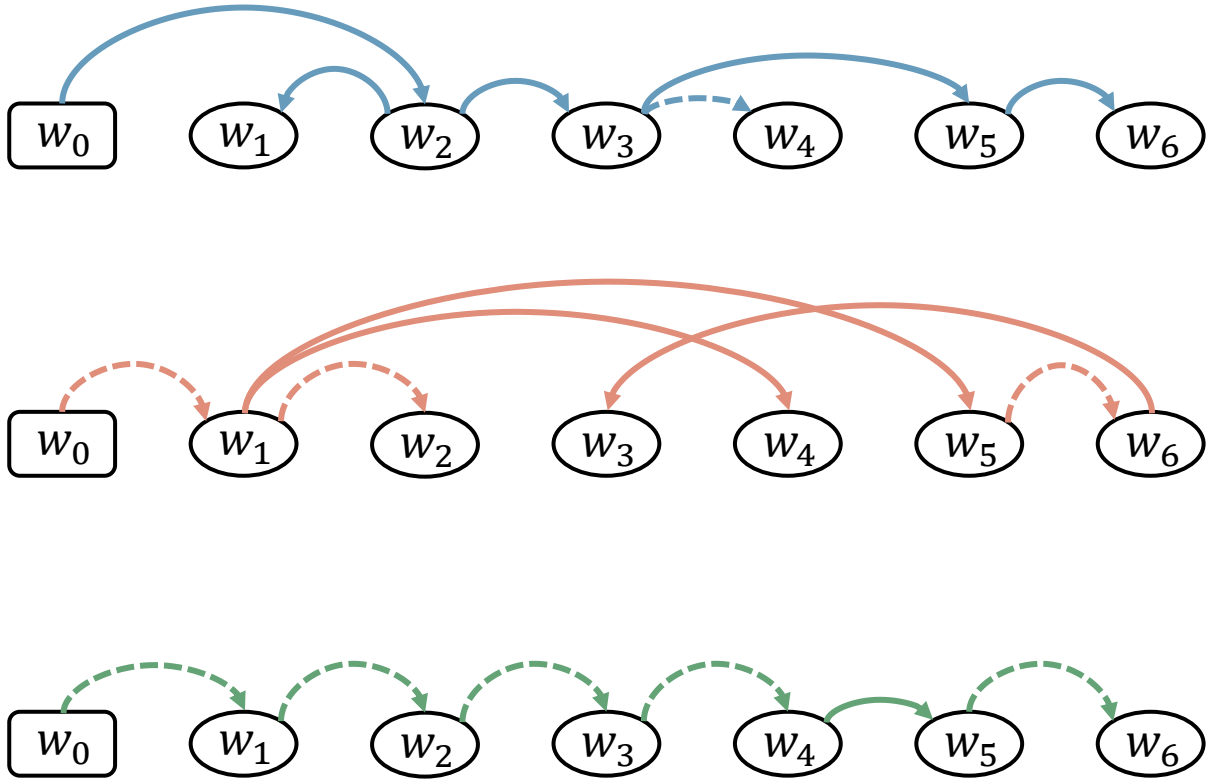
Step 2: Each plane is a tree. Create artificial heads to those nodes without incoming arcs.



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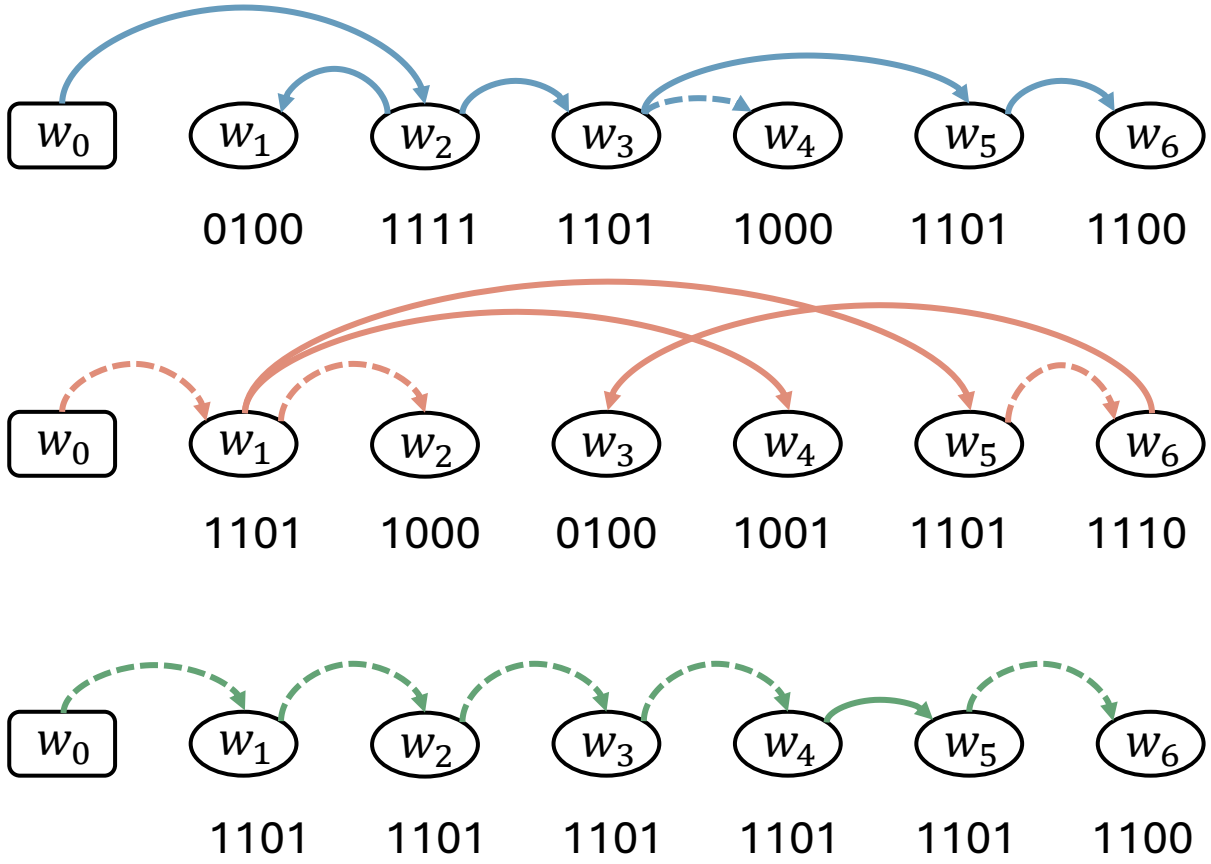
Step 3. Assign labels.



$4k$-bit Encoding

What about decoding?

$4k$-bit Encoding

- Same as bracketing-encoding.
- **L** for left arcs (**<**)and **R** right arcs (**/>**).
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$\mathbf{b_0b_1}$	**00**	Add i_0 to **L**.
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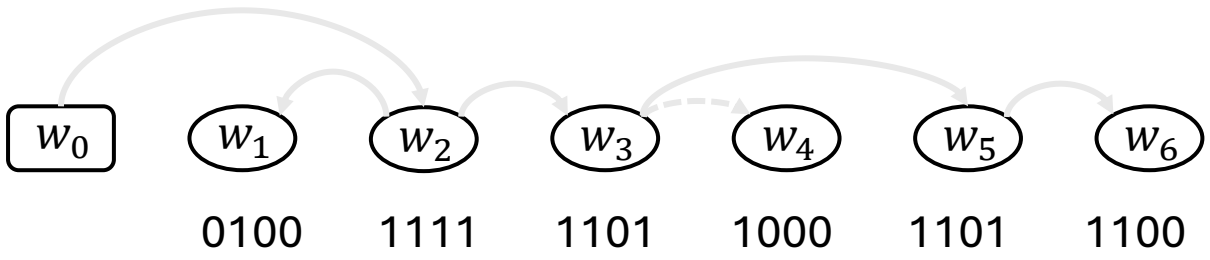
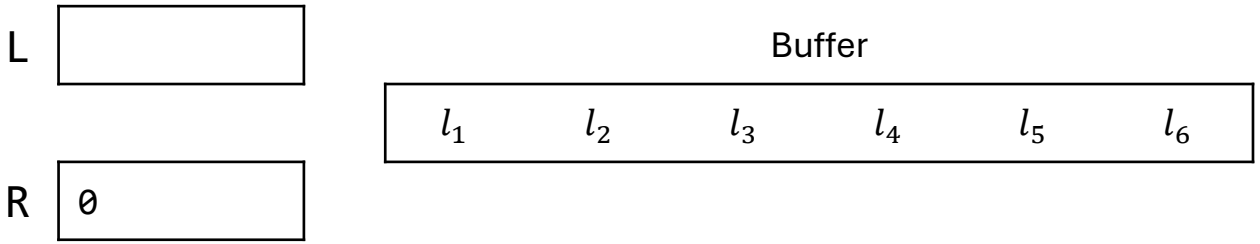


Initial state: Labels to buffer and **R** with the root dependency.



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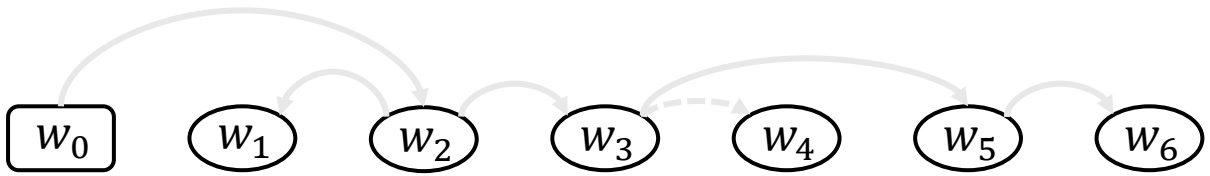

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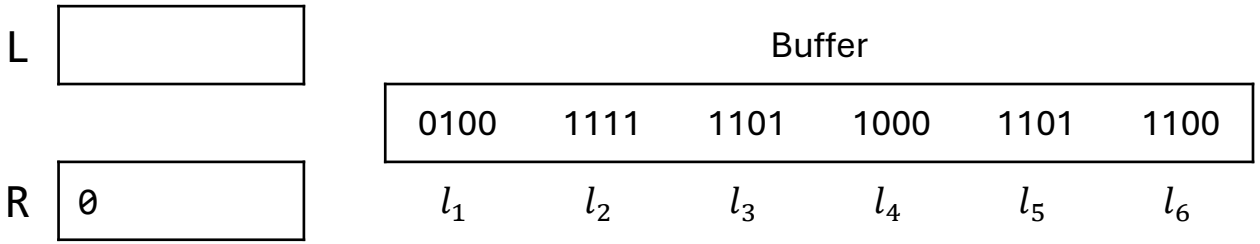

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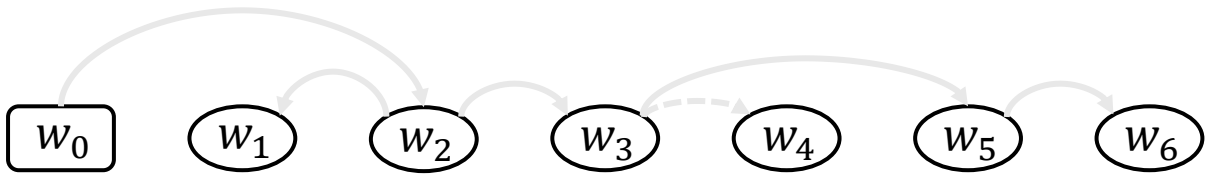

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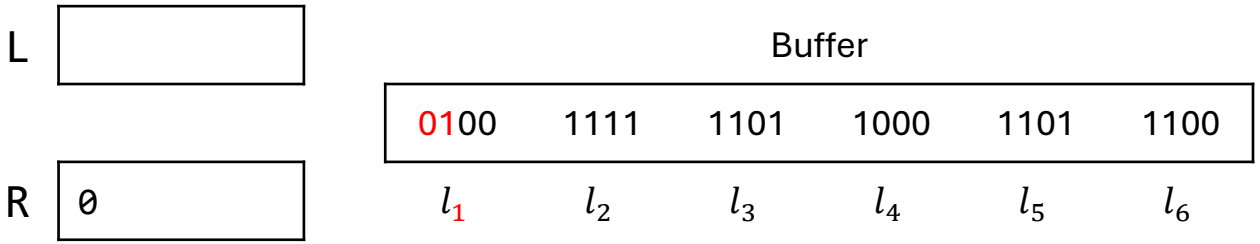

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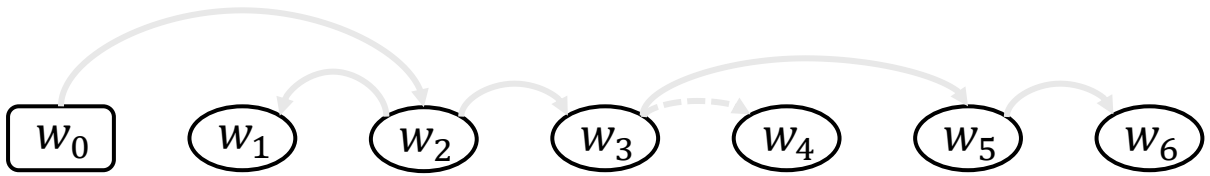

Initial state: Labels to buffer and **R** with the root dependency.

L 1_1

R 0

Buffer

0100	1111	1101	1000	1101	1100
l_1	l_2	l_3	l_4	l_5	l_6

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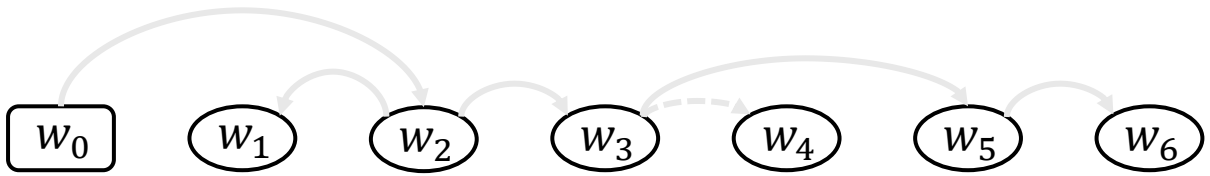

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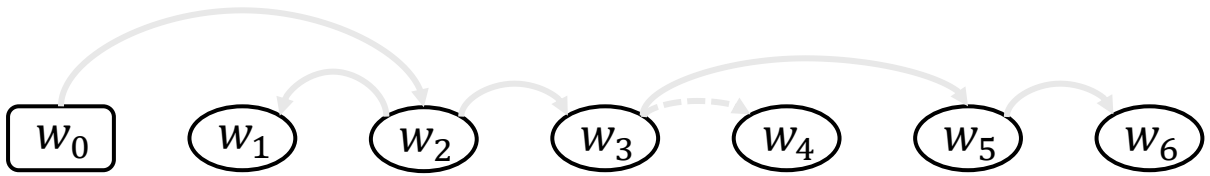

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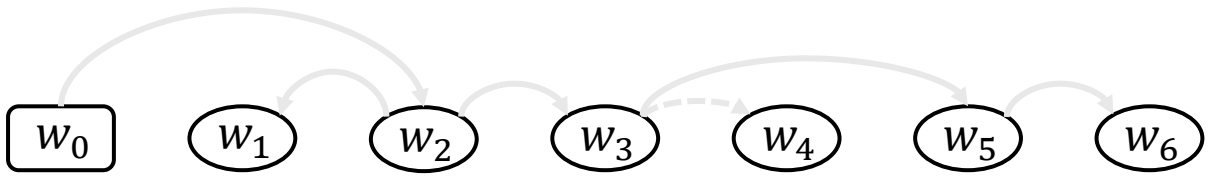

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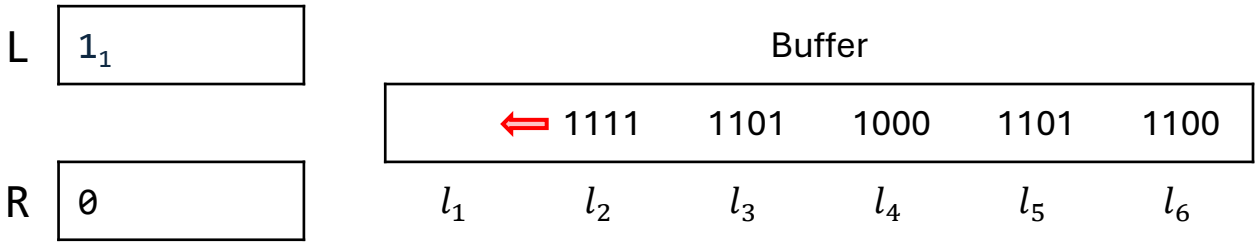

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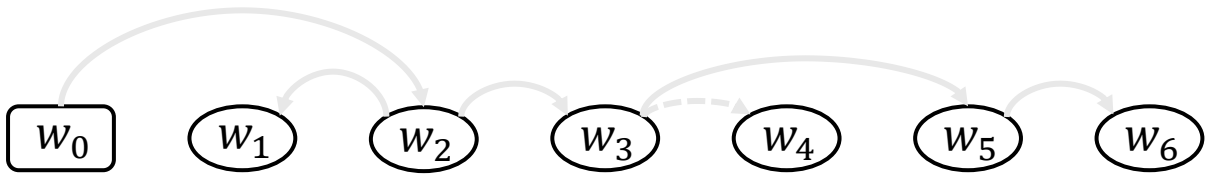

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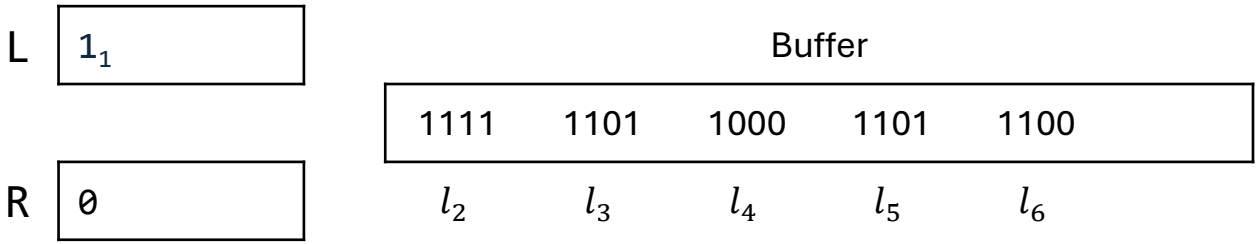

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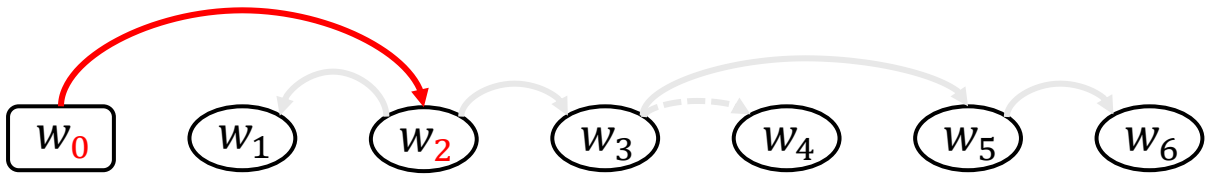

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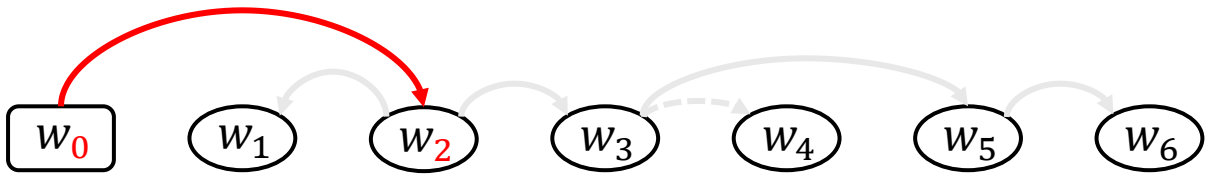

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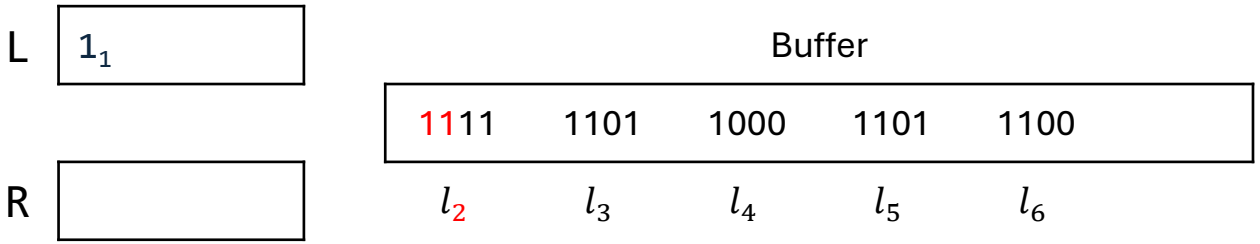

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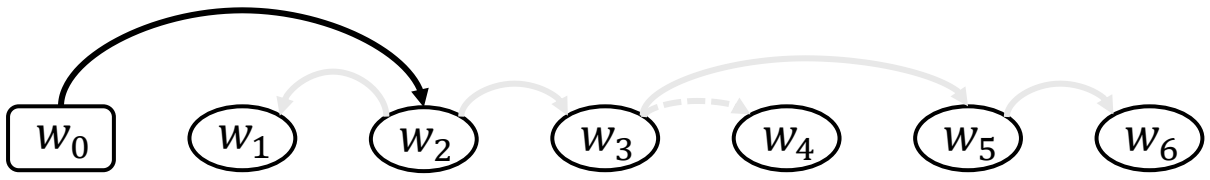

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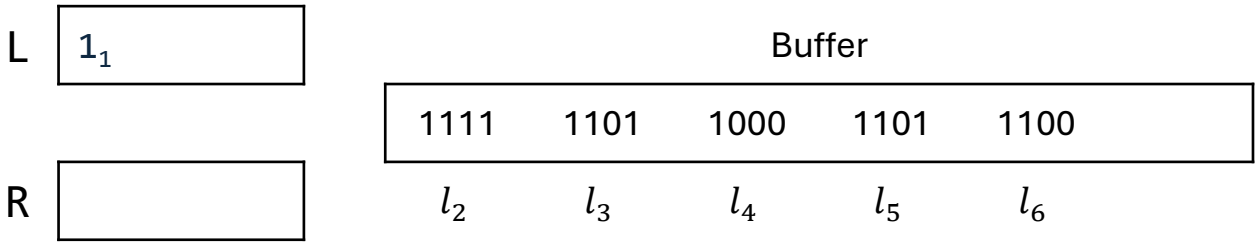

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	10	Resolve **R** $\rightarrow w_i$.
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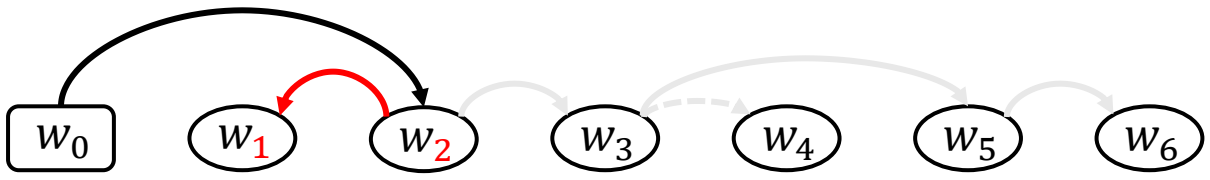
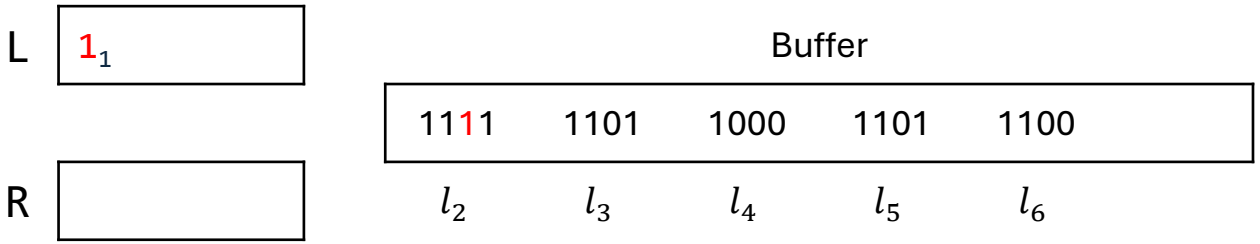
$\mathbf{b_2}$	**1**	Resolve $\mathbf{L_p} \leftarrow w_i$ and pop if **p**.
$\mathbf{b_3}$	**1**	Add i to **R**.



Initial state: Labels to buffer and **R** with the root dependency.



$4k$-bit Encoding

- Same as bracketing-encoding.
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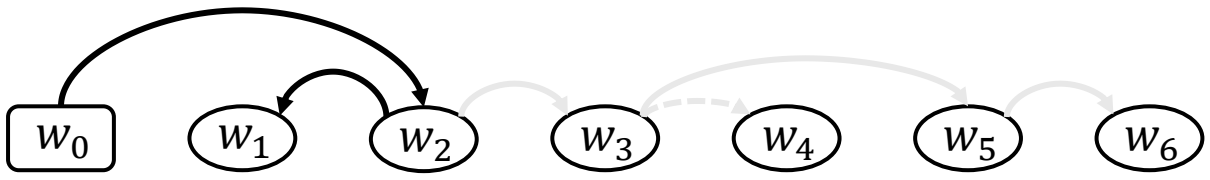
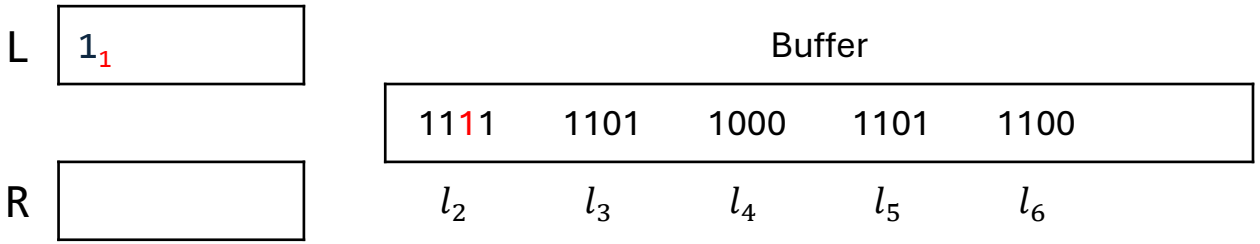


Initial state: Labels to buffer and **R** with the root dependency.



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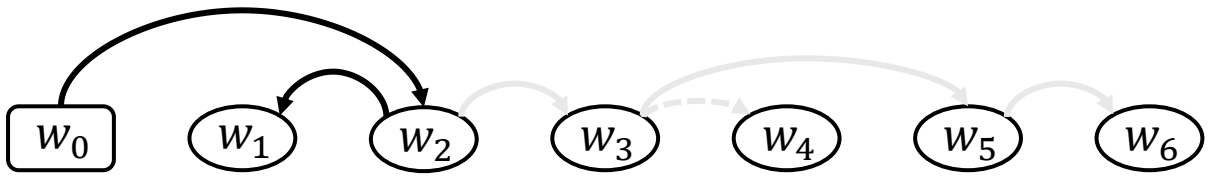

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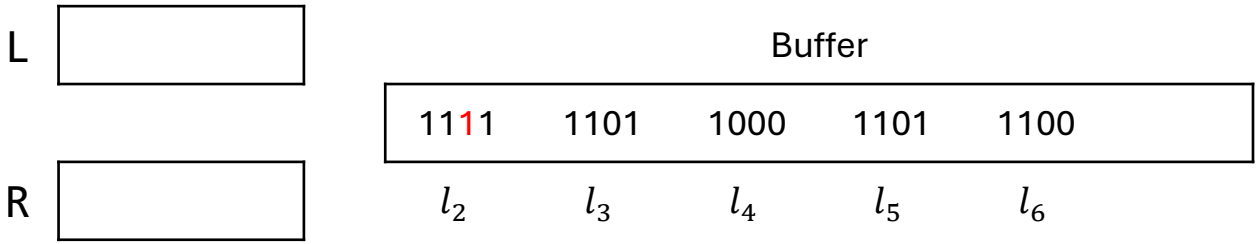

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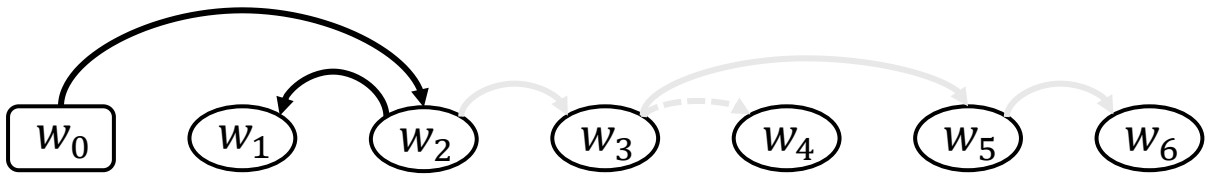

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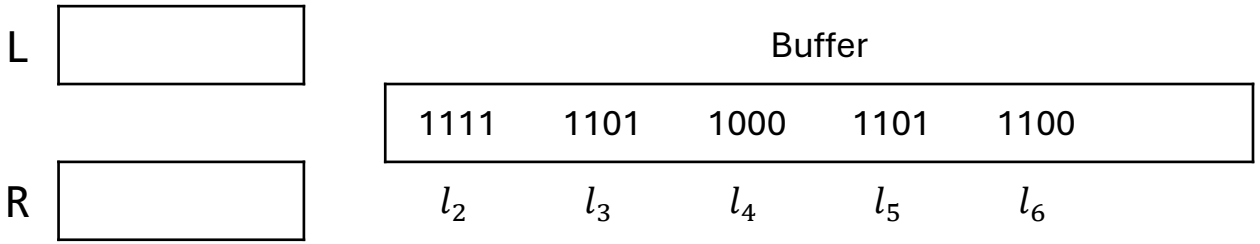

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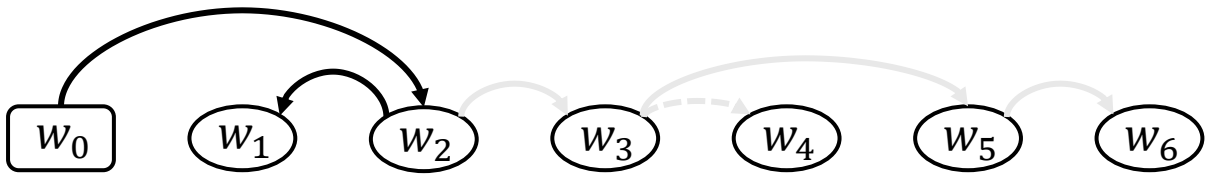

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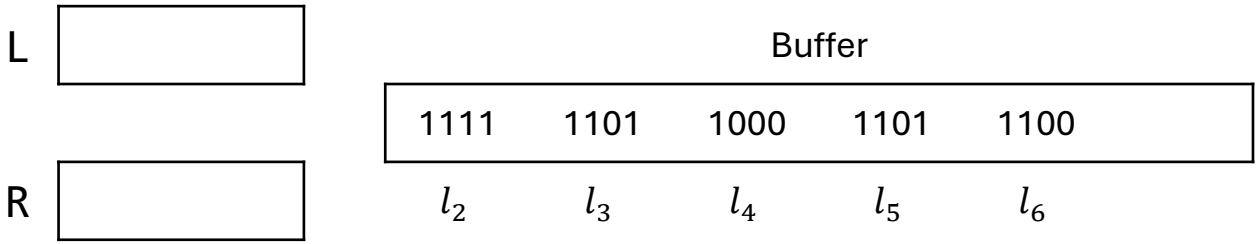

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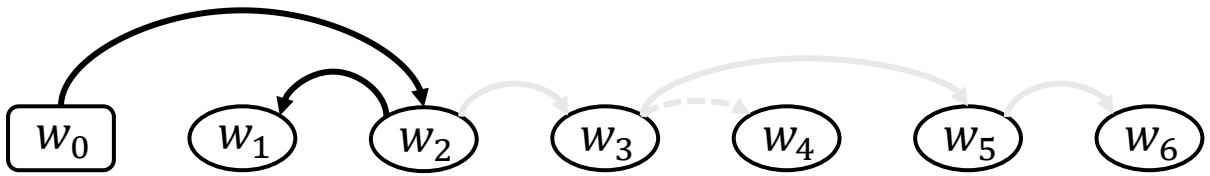

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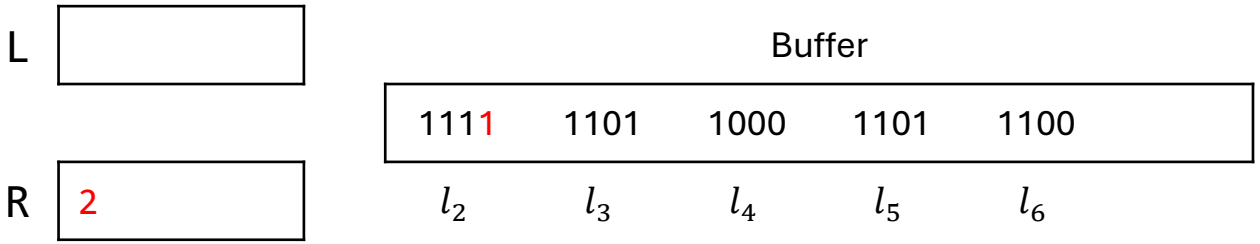

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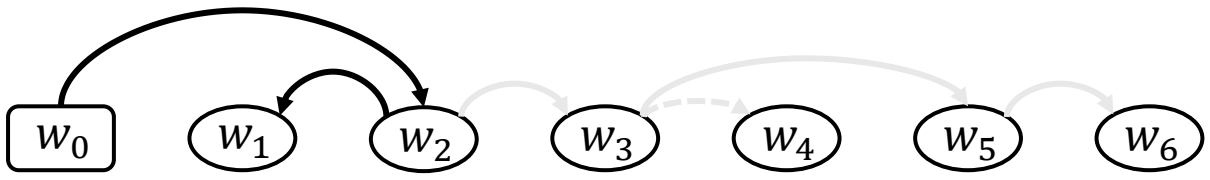

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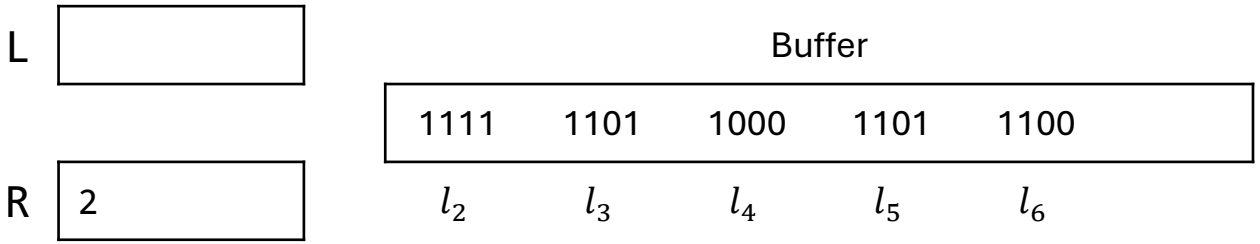

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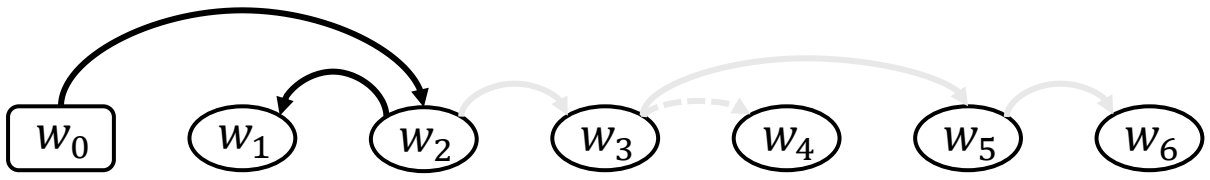

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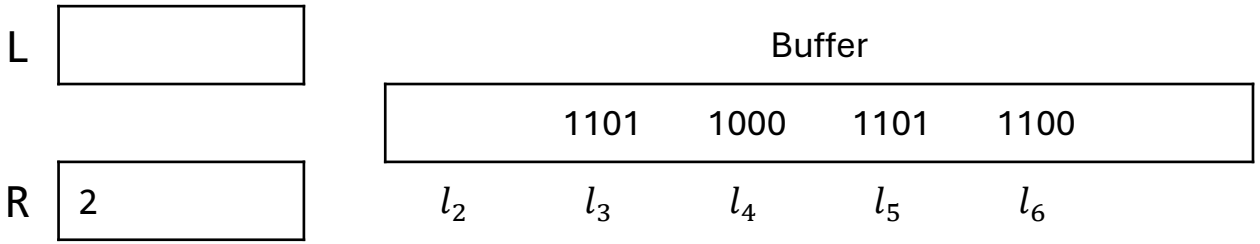

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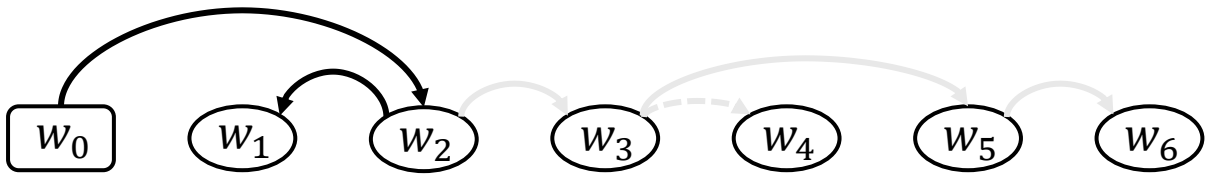

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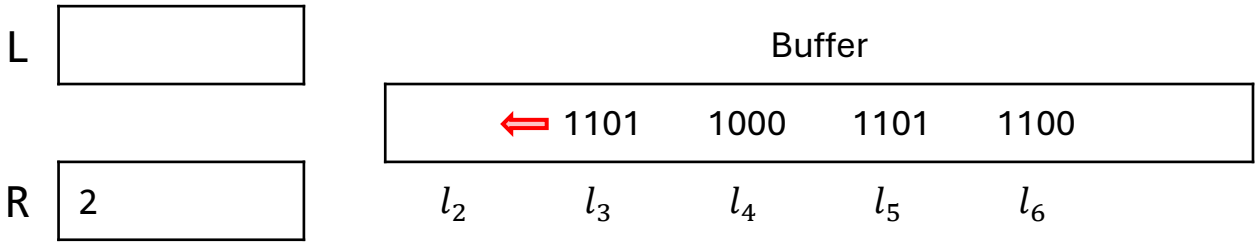

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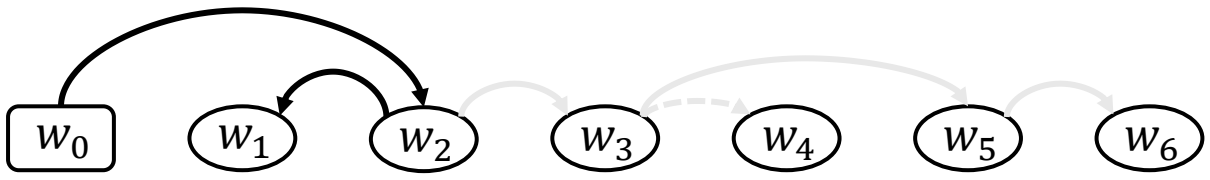

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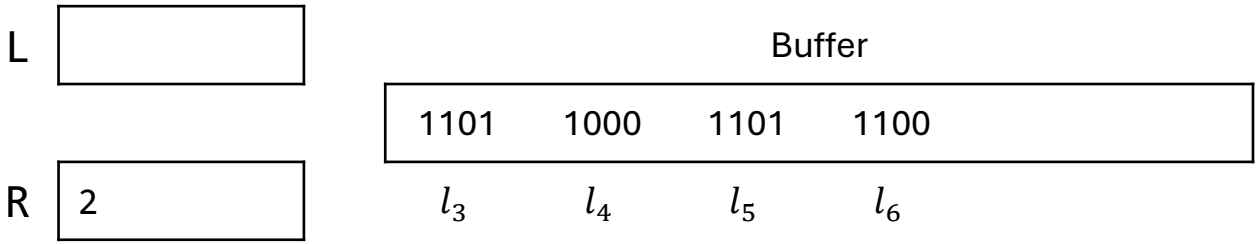

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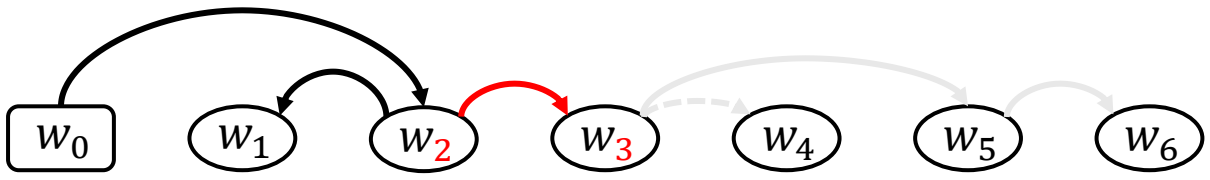

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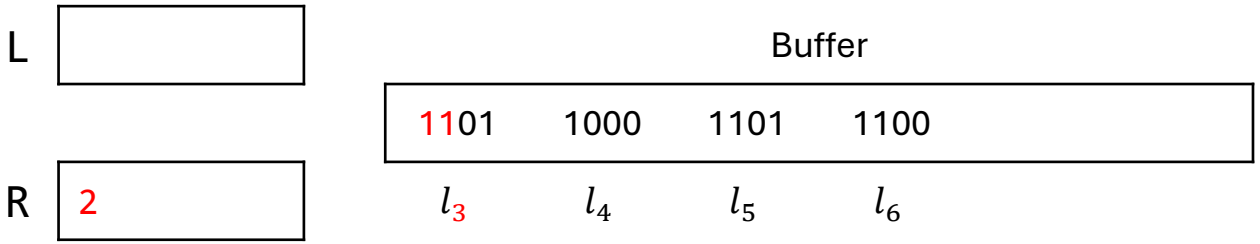

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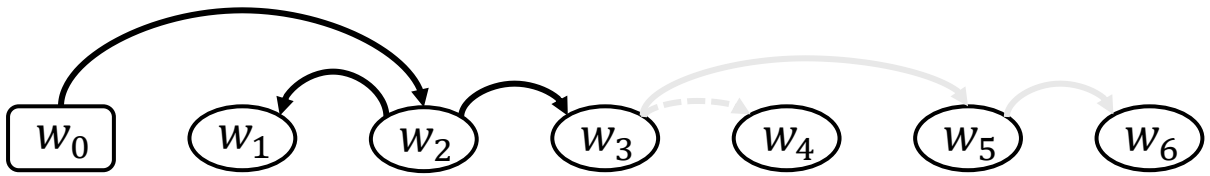

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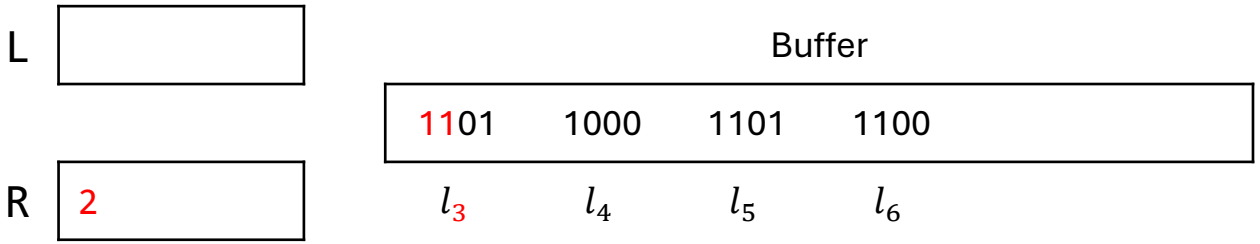

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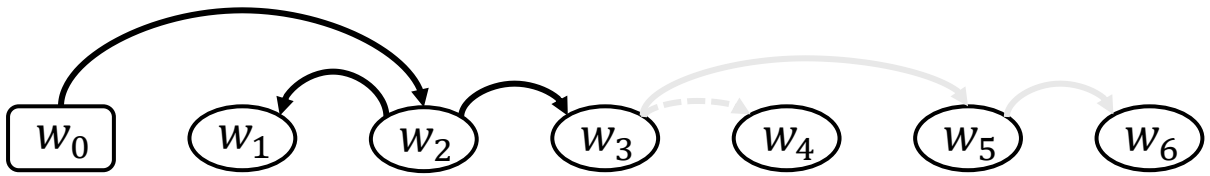

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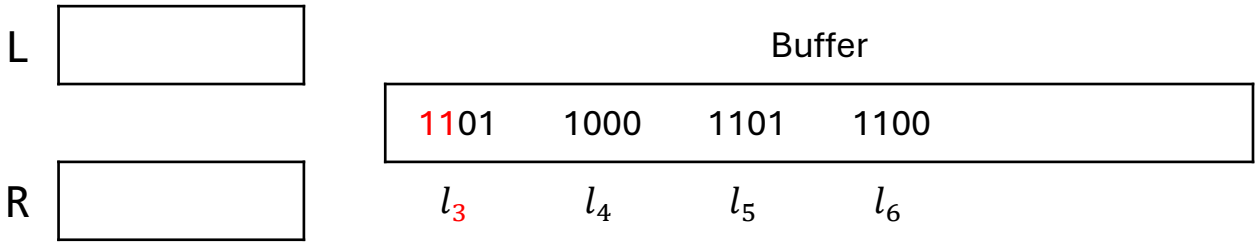

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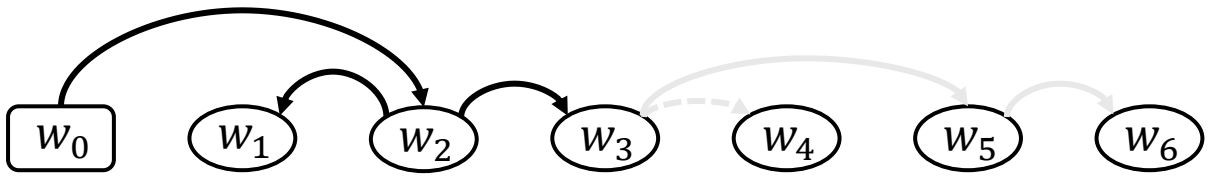

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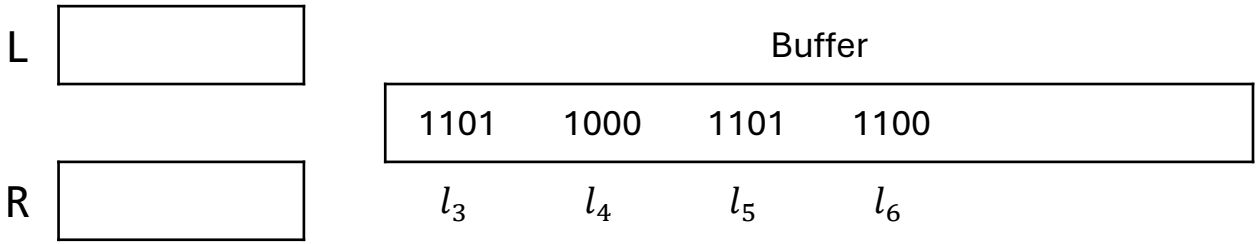

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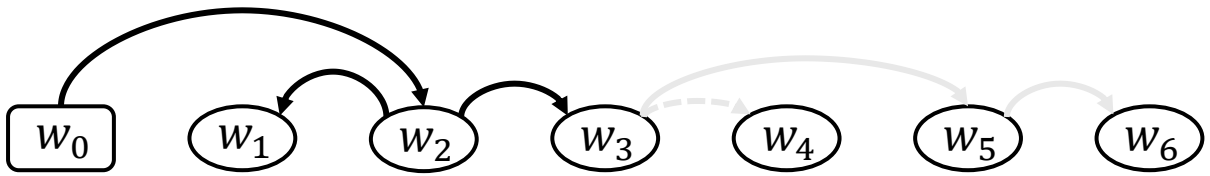

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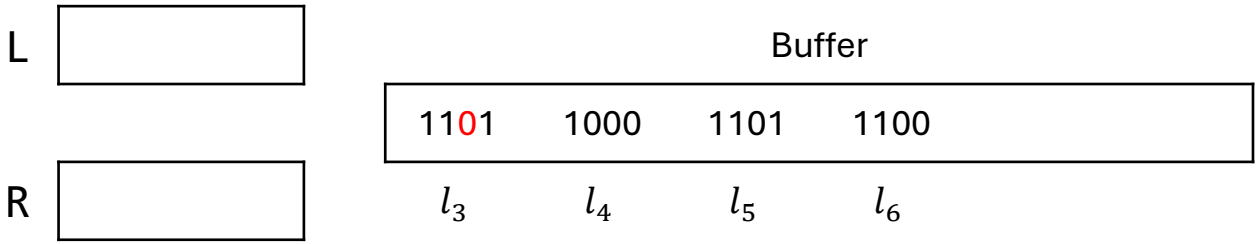

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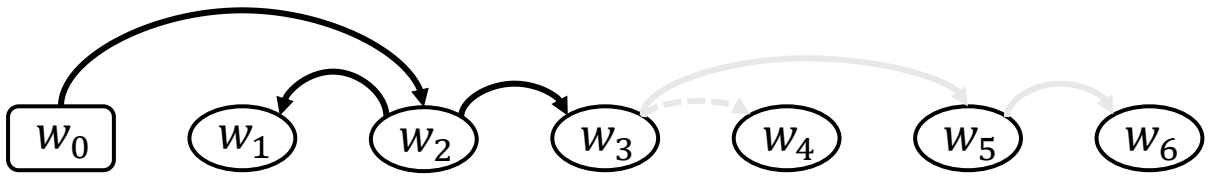

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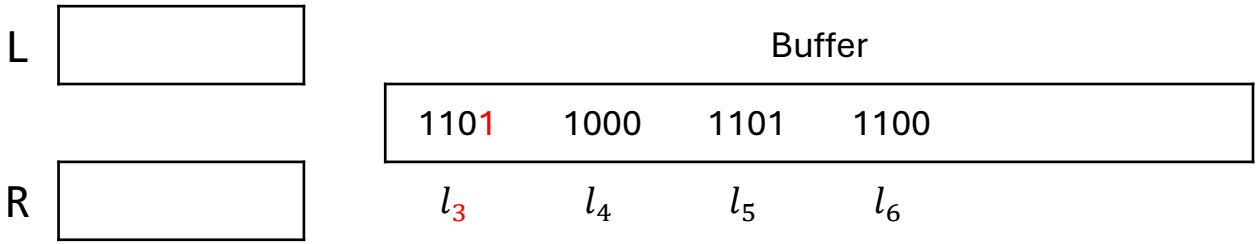

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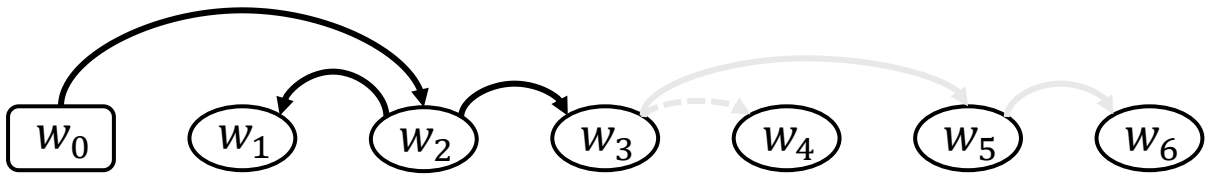

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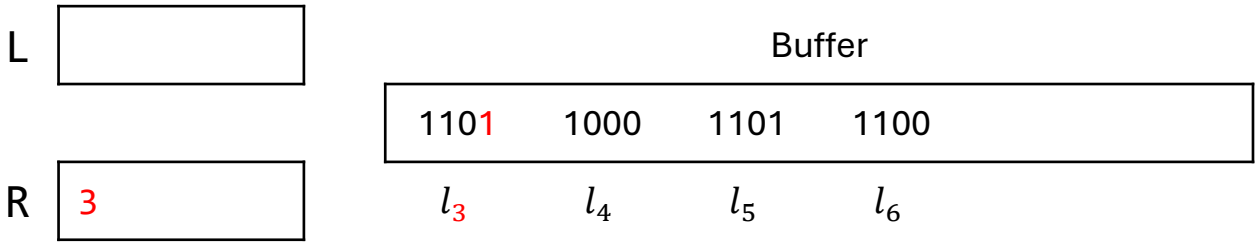

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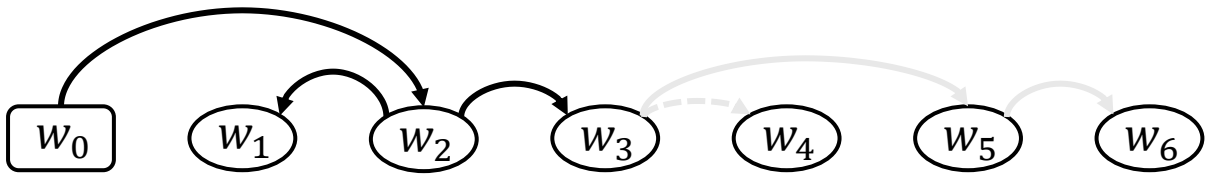
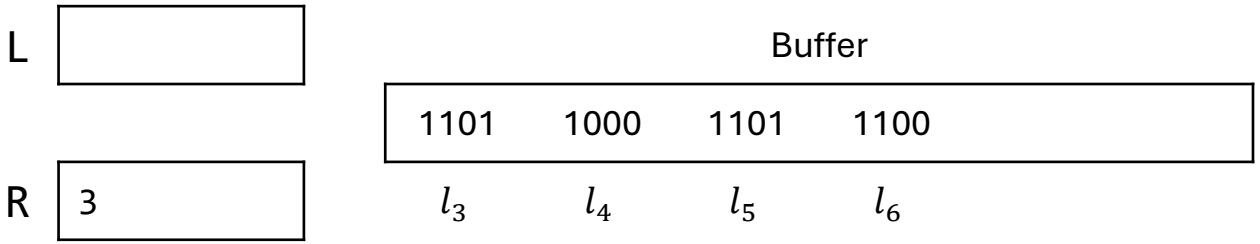


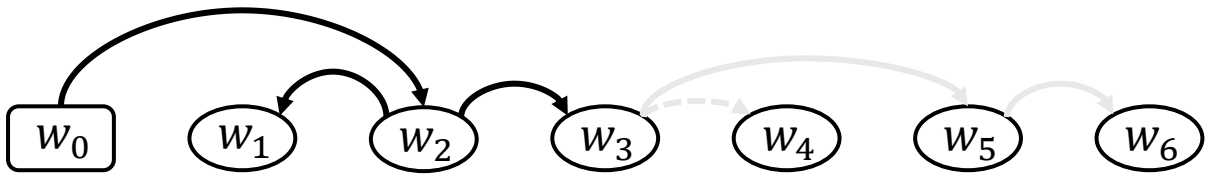
Initial state: Labels to buffer and **R** with the root dependency.



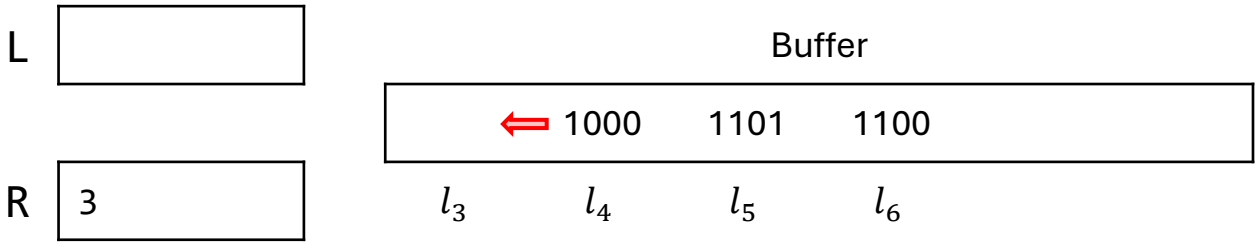
$4k$-bit Encoding

- Same as bracketing-encoding.
- **L** for left arcs (**<**)and **R** right arcs (**/>**).
- Store i and $\mathbf{b_1}$.

$\mathbf{b_0 b_1}$	**00**	Add i_0 to **L**.
	01	Add i_1 to **L**.
	10	Resolve **R** $\rightarrow w_i$.
	11	Resolve **R** $\rightarrow w_i$ and pop **R**.
$\mathbf{b_2}$	**1**	Resolve $\mathbf{L_p} \leftarrow w_i$ and pop if **p**.
$\mathbf{b_3}$	**1**	Add i to **R**.



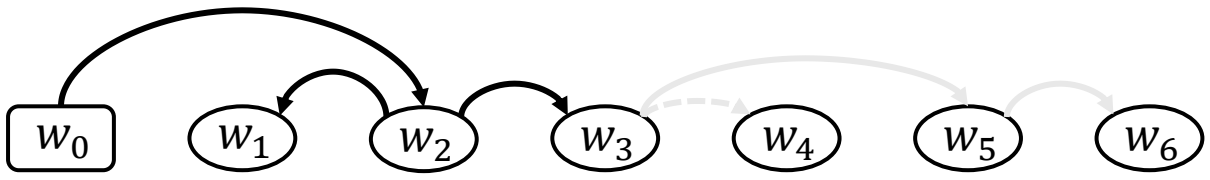
Initial state: Labels to buffer and **R** with the root dependency.



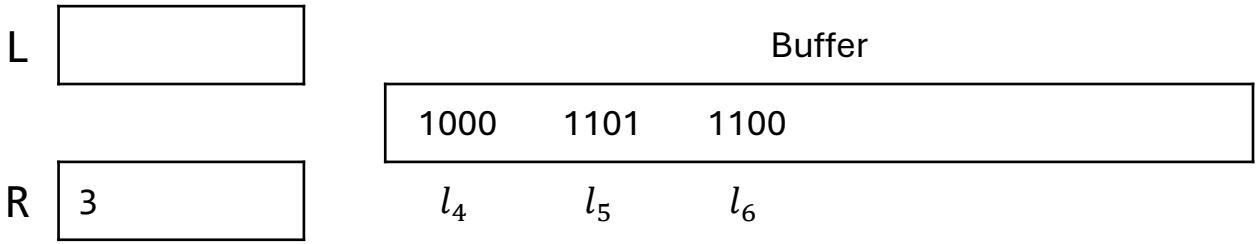
$4k$-bit Encoding

- Same as bracketing-encoding.
- **L** for left arcs (**<**)and **R** right arcs (**/>**).
- Store i and $\mathbf{b_1}$.

$\mathbf{b_0 b_1}$	**00**	Add i_0 to **L**.
	01	Add i_1 to **L**.
	10	Resolve **R** $\rightarrow w_i$.
	11	Resolve **R** $\rightarrow w_i$ and pop **R**.
$\mathbf{b_2}$	**1**	Resolve $\mathbf{L_p} \leftarrow w_i$ and pop if **p**.
$\mathbf{b_3}$	**1**	Add i to **R**.



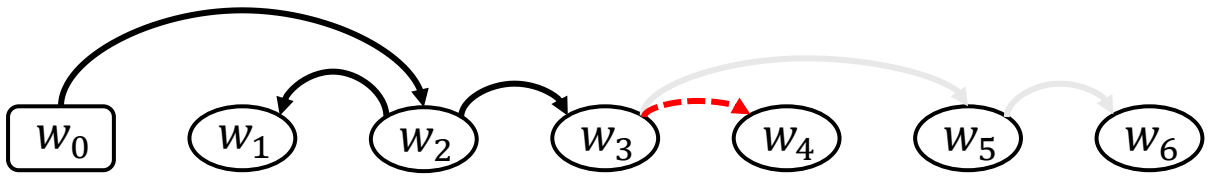
Initial state: Labels to buffer and **R** with the root dependency.



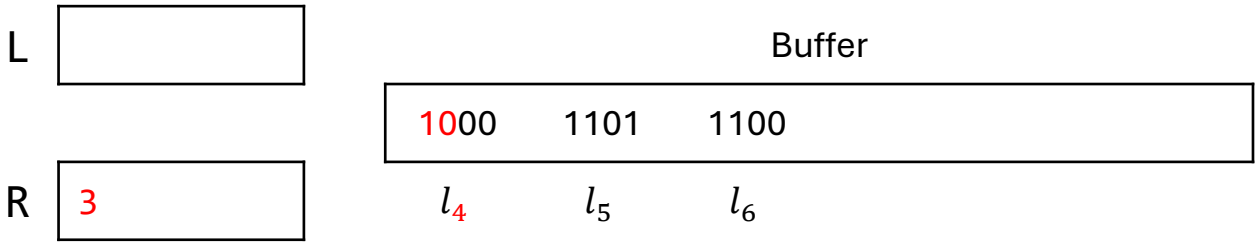
$4k$-bit Encoding

- Same as bracketing-encoding.
- **L** for left arcs (**<**)and **R** right arcs (**/>**).
- Store i and $\mathbf{b_1}$.

$\mathbf{b_0 b_1}$	**00**	Add i_0 to **L**.
	01	Add i_1 to **L**.
	10	Resolve **R** → w_i.
	11	Resolve **R** → w_i and pop **R**.
$\mathbf{b_2}$	**1**	Resolve $\mathbf{L_p}$ ← w_i and pop if **p**.
$\mathbf{b_3}$	**1**	Add i to **R**.



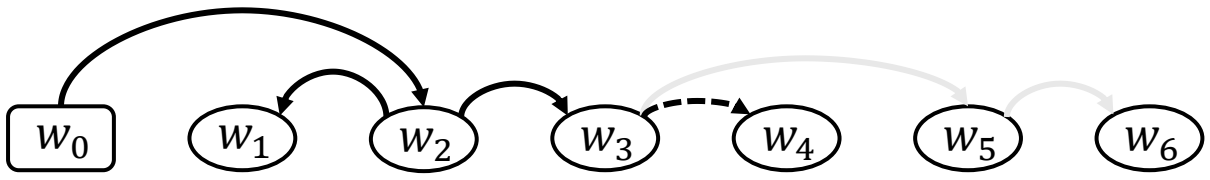
Initial state: Labels to buffer and **R** with the root dependency.



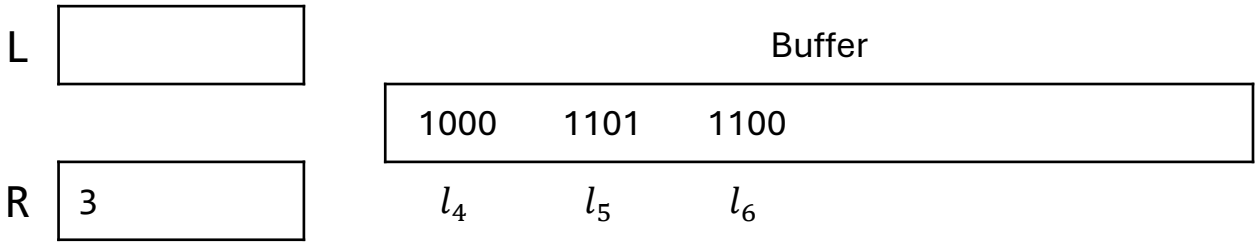
$4k$-bit Encoding

- Same as bracketing-encoding.
- **L** for left arcs (**<**)and **R** right arcs (**/>**).
- Store i and $\mathbf{b_1}$.

$\mathbf{b_0 b_1}$	**00**	Add i_0 to **L**.
	01	Add i_1 to **L**.
	10	Resolve **R** $\rightarrow w_i$.
	11	Resolve **R** $\rightarrow w_i$ and pop **R**.
$\mathbf{b_2}$	**1**	Resolve $\mathbf{L_p} \leftarrow w_i$ and pop if **p**.
$\mathbf{b_3}$	**1**	Add i to **R**.



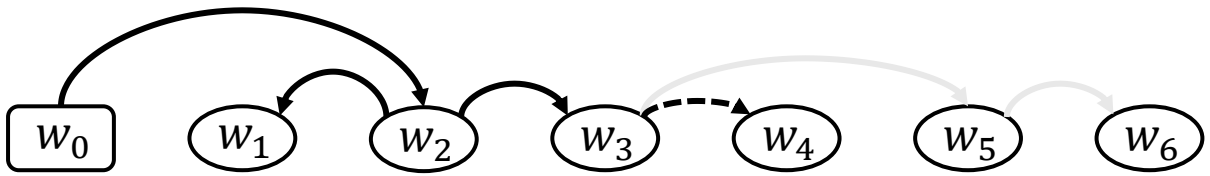
Initial state: Labels to buffer and **R** with the root dependency.



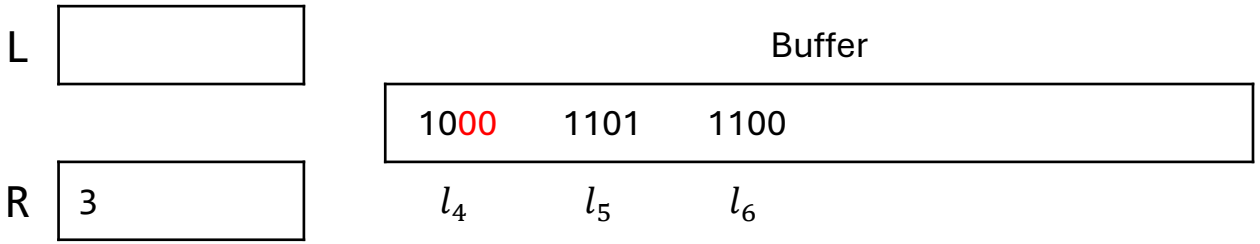
$4k$-bit Encoding

- Same as bracketing-encoding.
- **L** for left arcs (**<**)and **R** right arcs (**/>**).
- Store i and $\mathbf{b_1}$.

$\mathbf{b_0b_1}$	**00**	Add i_0 to **L**.
	01	Add i_1 to **L**.
	10	Resolve **R** $\rightarrow w_i$.
	11	Resolve **R** $\rightarrow w_i$ and pop **R**.
$\mathbf{b_2}$	**1**	Resolve $\mathbf{L_p} \leftarrow w_i$ and pop if **p**.
$\mathbf{b_3}$	**1**	Add i to **R**.



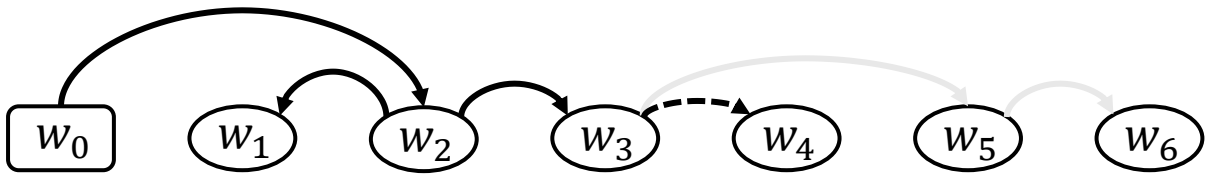
Initial state: Labels to buffer and **R** with the root dependency.



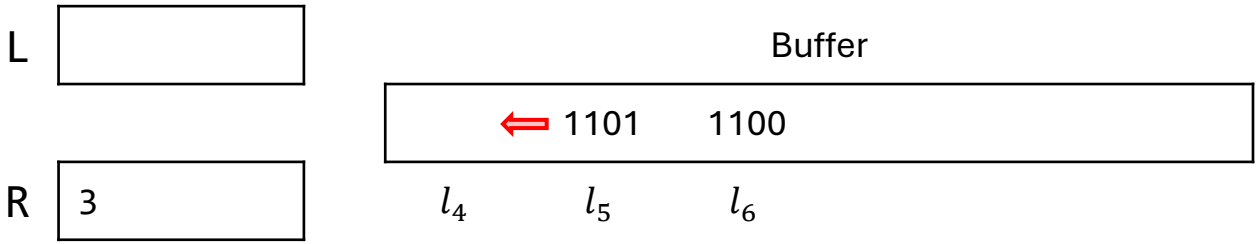
$4k$-bit Encoding

- Same as bracketing-encoding.
- **L** for left arcs (**<**)and **R** right arcs (**/>**).
- Store i and $\mathbf{b_1}$.

$\mathbf{b_0b_1}$	**00**	Add i_0 to **L**.
	01	Add i_1 to **L**.
	10	Resolve **R** $\rightarrow w_i$.
	11	Resolve **R** $\rightarrow w_i$ and pop **R**.
$\mathbf{b_2}$	**1**	Resolve $\mathbf{L_p} \leftarrow w_i$ and pop if **p**.
$\mathbf{b_3}$	**1**	Add i to **R**.



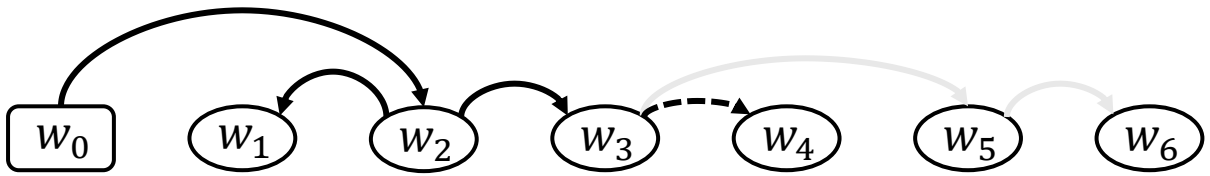
Initial state: Labels to buffer and **R** with the root dependency.



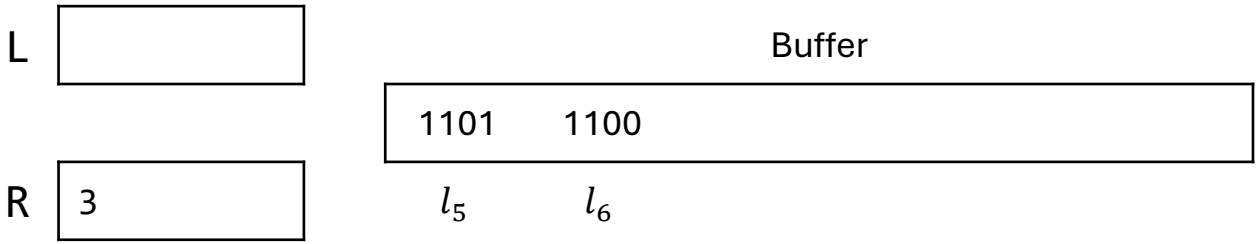
$4k$-bit Encoding

- Same as bracketing-encoding.
- **L** for left arcs (**<**)and **R** right arcs (**/>**).
- Store i and $\mathbf{b_1}$.

$\mathbf{b_0b_1}$	**00**	Add i_0 to **L**.
	01	Add i_1 to **L**.
	10	Resolve **R** $\rightarrow w_i$.
	11	Resolve **R** $\rightarrow w_i$ and pop **R**.
$\mathbf{b_2}$	**1**	Resolve $\mathbf{L_p} \leftarrow w_i$ and pop if **p**.
$\mathbf{b_3}$	**1**	Add i to **R**.



Initial state: Labels to buffer and **R** with the root dependency.



$4k$-bit Encoding

- Same as bracketing-encoding.
- **L** for left arcs (**<**)and **R** right arcs (**/>**).
- Store i and $\mathbf{b_1}$.

$\mathbf{b_0b_1}$	**00**	Add i_0 to **L**.
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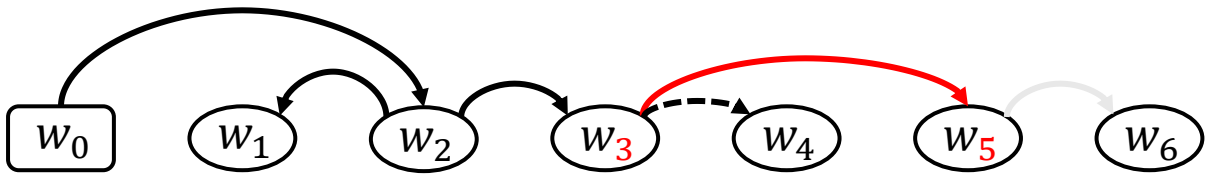
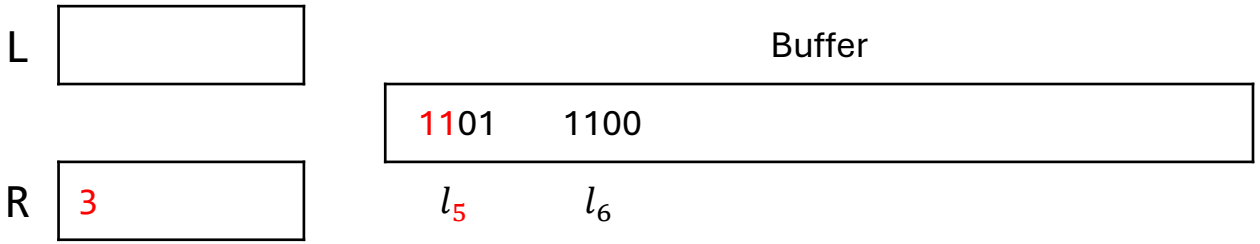
	10	Resolve **R** $\rightarrow w_i$.
	11	Resolve **R** $\rightarrow w_i$ and pop **R**.
$\mathbf{b_2}$	**1**	Resolve $\mathbf{L_p} \leftarrow w_i$ and pop if **p**.
$\mathbf{b_3}$	**1**	Add i to **R**.



Initial state: Labels to buffer and **R** with the root dependency.



$4k$-bit Encoding

- Same as bracketing-encoding.
- **L** for left arcs (**<**)and **R** right arcs (**/>**).
- Store i and $\mathbf{b_1}$.

$\mathbf{b_0 b_1}$	**00**	Add i_0 to **L**.
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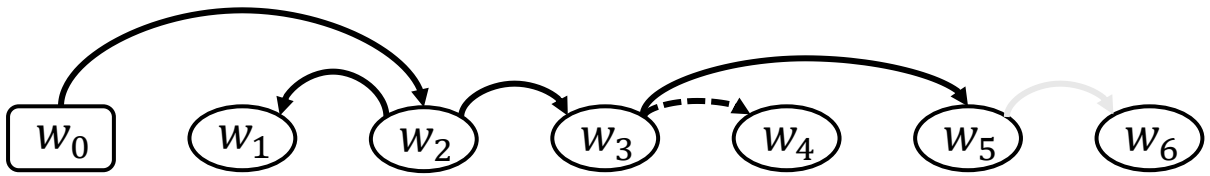
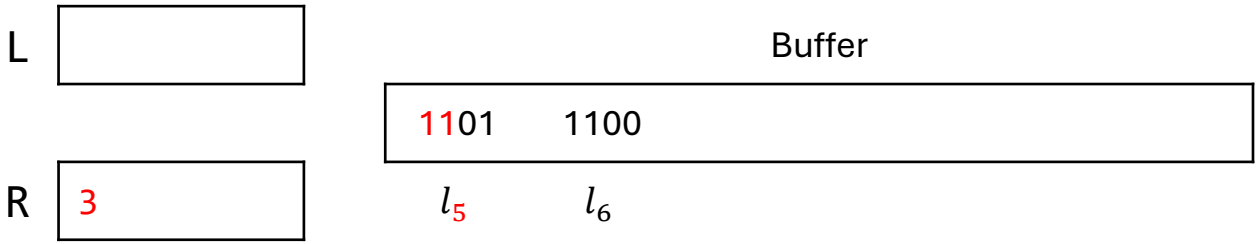
	10	Resolve **R** $\rightarrow w_i$.
	11	Resolve **R** $\rightarrow w_i$ and pop **R**.
$\mathbf{b_2}$	**1**	Resolve $\mathbf{L_p} \leftarrow w_i$ and pop if **p**.
$\mathbf{b_3}$	**1**	Add i to **R**.



Initial state: Labels to buffer and **R** with the root dependency.



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- Same as bracketing-encoding.
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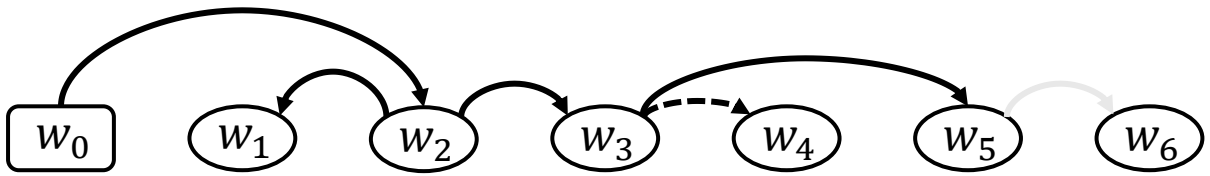
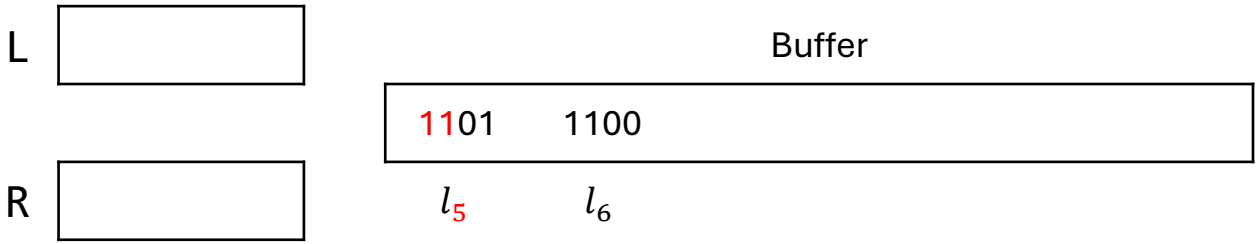
	11	Resolve **R** $\rightarrow w_i$ and pop **R**.
$\mathbf{b_2}$	**1**	Resolve $\mathbf{L_p} \leftarrow w_i$ and pop if **p**.
$\mathbf{b_3}$	**1**	Add i to **R**.



Initial state: Labels to buffer and **R** with the root dependency.



$4k$-bit Encoding

- Same as bracketing-encoding.
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$\mathbf{b_0b_1}$	**00**	Add i_0 to **L**.
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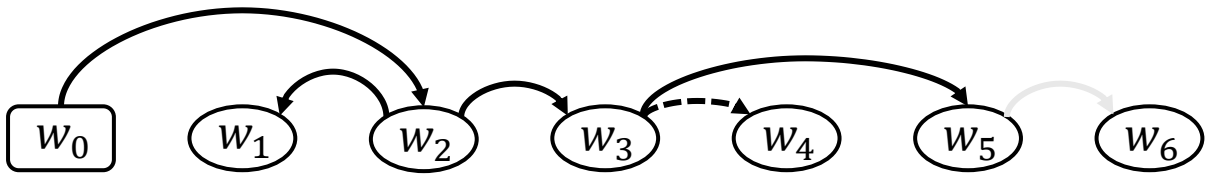
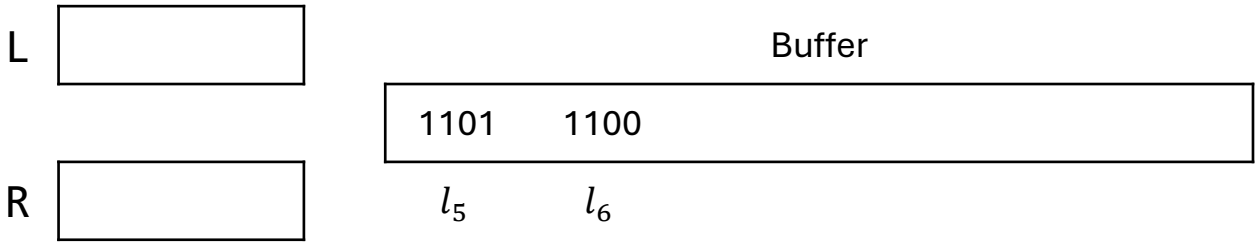


Initial state: Labels to buffer and **R** with the root dependency.



$4k$-bit Encoding

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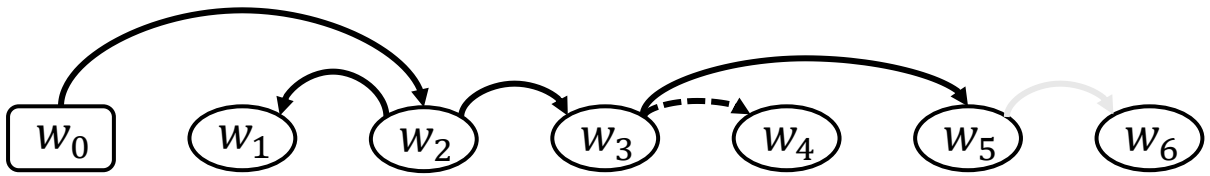
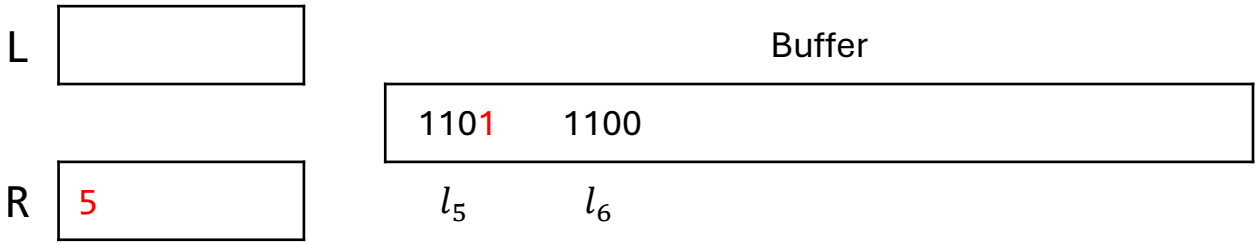


Initial state: Labels to buffer and **R** with the root dependency.



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$\mathbf{b_2}$	**1**	Resolve $\mathbf{L_p} \leftarrow w_i$ and pop if **p**.
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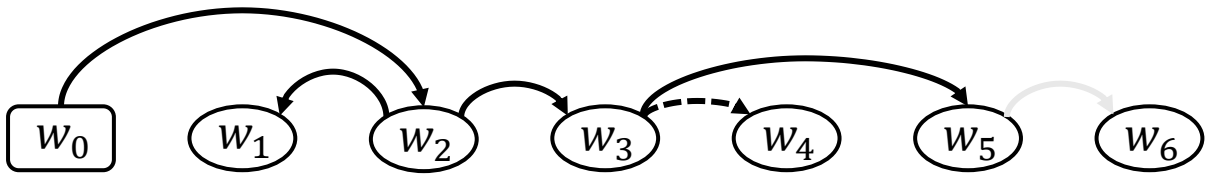
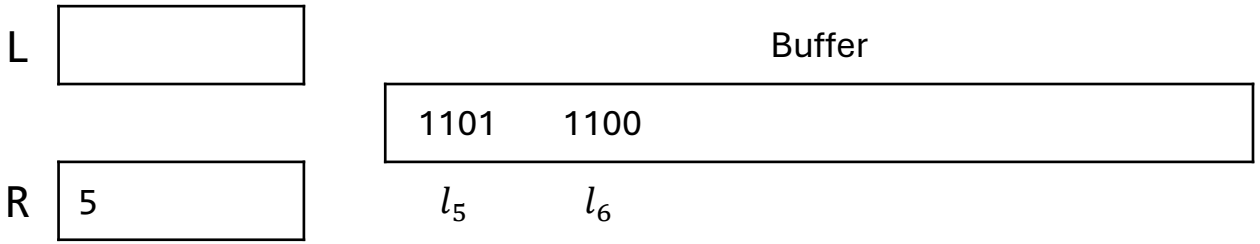


Initial state: Labels to buffer and **R** with the root dependency.



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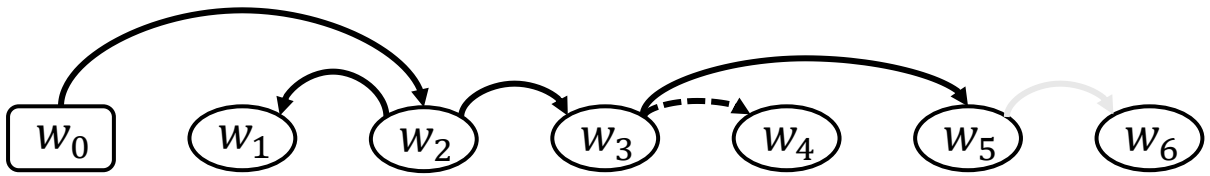
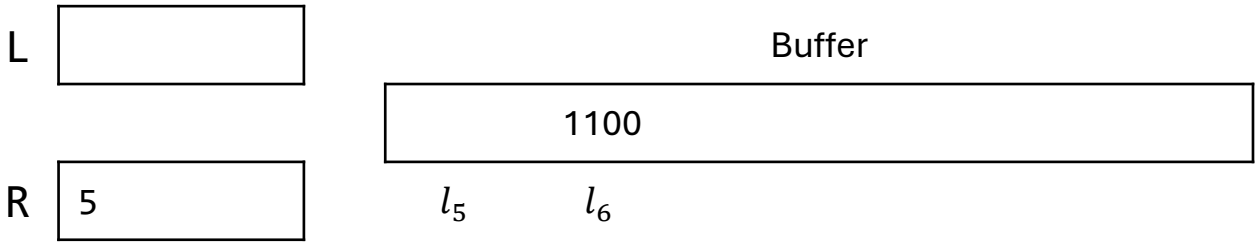


Initial state: Labels to buffer and **R** with the root dependency.



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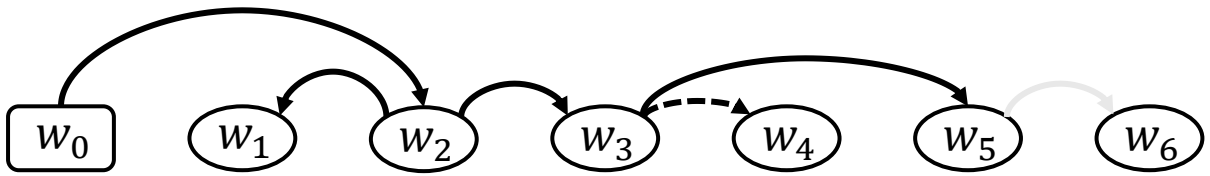
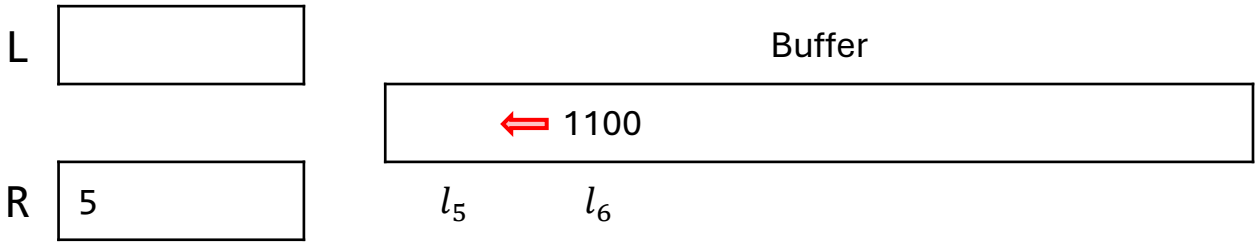


Initial state: Labels to buffer and **R** with the root dependency.



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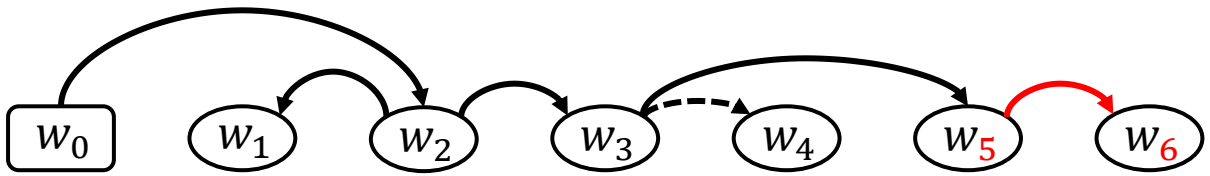

Initial state: Labels to buffer and **R** with the root dependency.

$4k$-bit Encoding

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$\mathbf{b_0b_1}$	**00**	Add i_0 to **L**.
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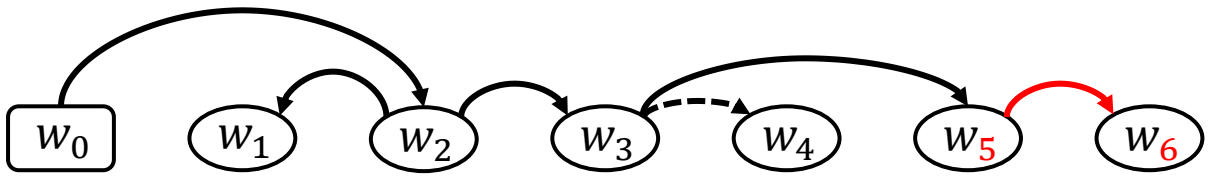

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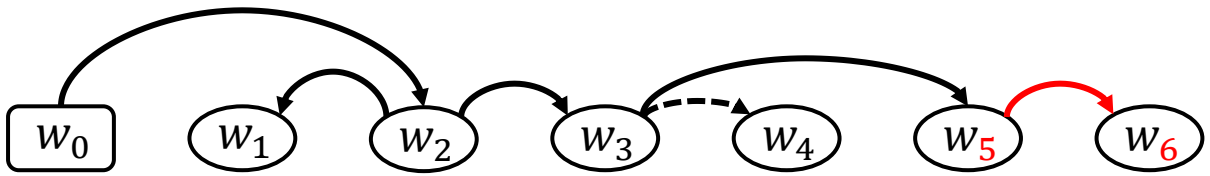

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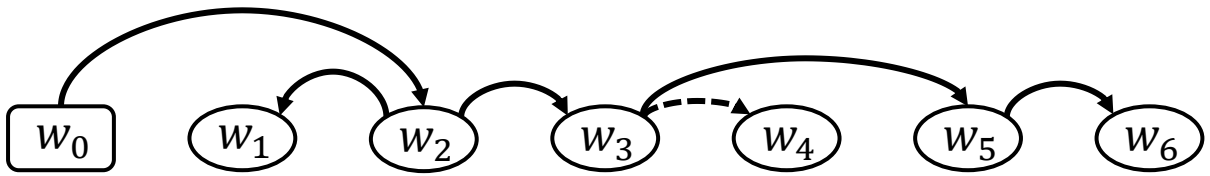
- Store i and $\mathbf{b_1}$.

$\mathbf{b_0 b_1}$	**00**	Add i_0 to **L**.
	01	Add i_1 to **L**.
	10	Resolve **R** $\rightarrow w_i$.
	11	Resolve **R** $\rightarrow w_i$ and pop **R**.
$\mathbf{b_2}$	**1**	Resolve $\mathbf{L_p} \leftarrow w_i$ and pop if **p**.
$\mathbf{b_3}$	**1**	Add i to **R**.



Initial state: Labels to buffer and **R** with the root dependency.

$4k$-bit Encoding

- Same as bracketing-encoding.
- **L** for left arcs (**<**)and **R** right arcs (**/>**).
- Store i and $\mathbf{b_1}$.

$\mathbf{b_0 b_1}$	**00**	Add i_0 to **L**.
	01	Add i_1 to **L**.
	10	Resolve **R** $\rightarrow w_i$.
	11	Resolve **R** $\rightarrow w_i$ and pop **R**.
$\mathbf{b_2}$	**1**	Resolve $\mathbf{L_p} \leftarrow w_i$ and pop if **p**.
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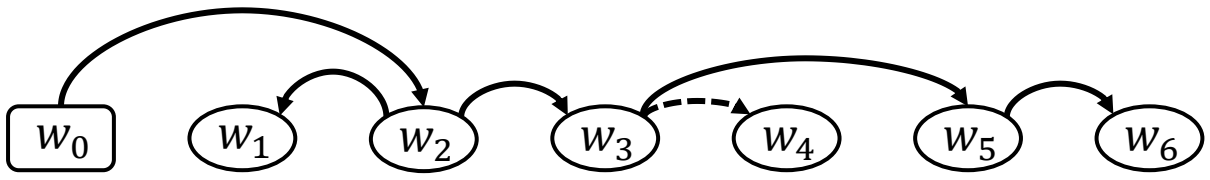
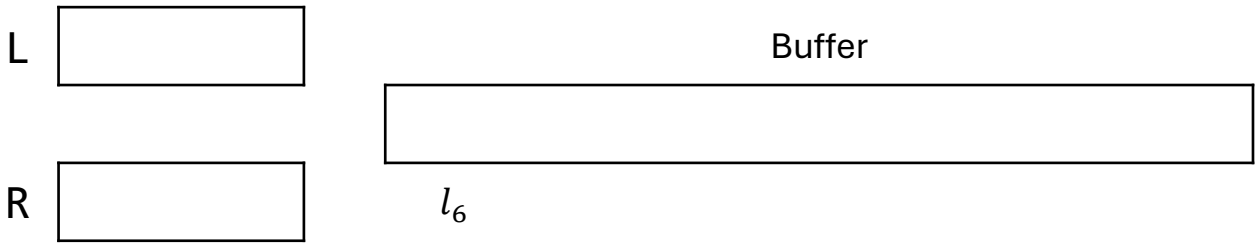


Finished!



$6k$-bit Encoding

Assumptions:

- Distribute the arcs in **relaxed 1-planar graphs**.
- For each node limit one head per direction.
- Encode each label ℓ_i with 6 bits: $\mathbf{b_0 b_1 b_2 b_3 b_4 b_5}$.

$\mathbf{b_0}$	w_i has a left head.
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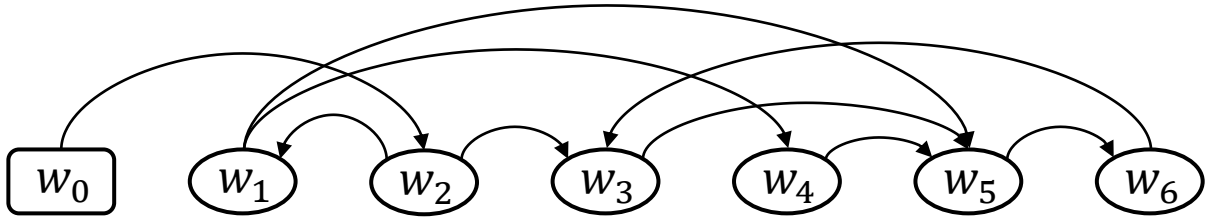
Step 1: Distribute the arcs in **relaxed 1-planar graphs.**



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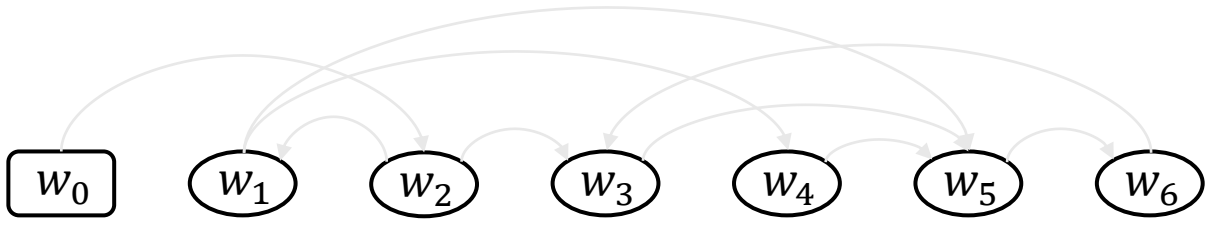

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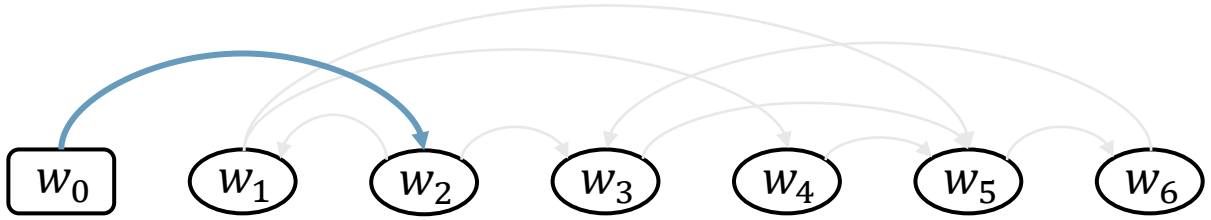

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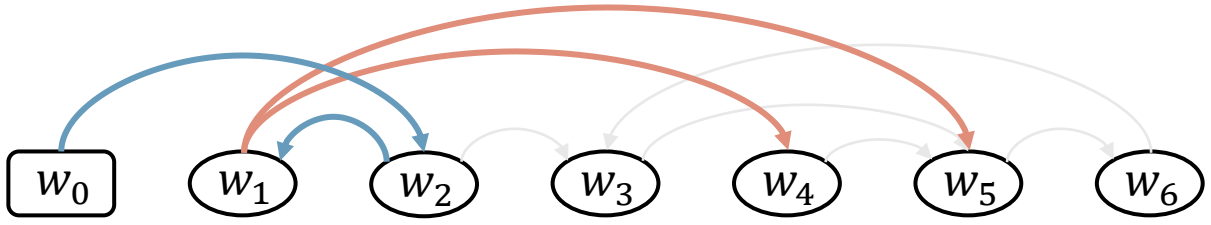

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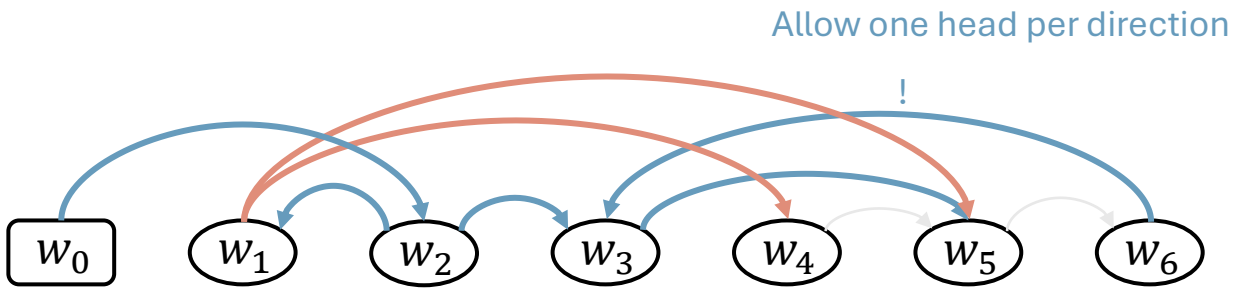

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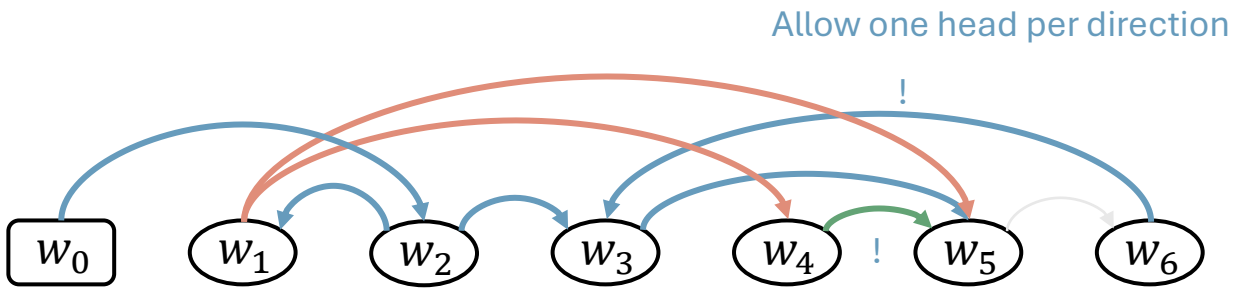

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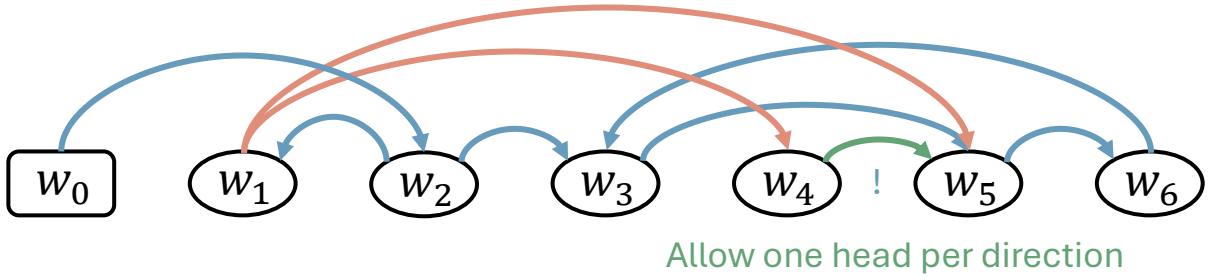

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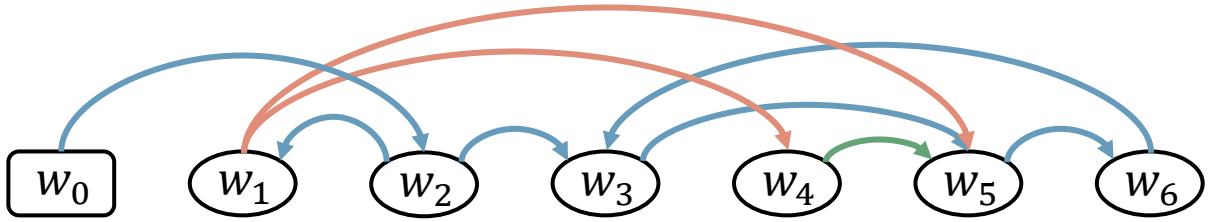
Step 2: Assign labels.



P1	000110	001001	111110	000000	111000	110001
P2	001000	000000	000000	100000	110000	000000
P3	000000	000000	000000	001000	110000	000000

$6k$-bit Encoding

Assumptions:

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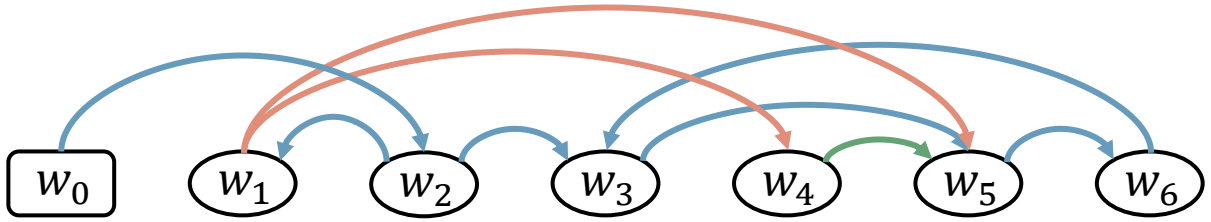
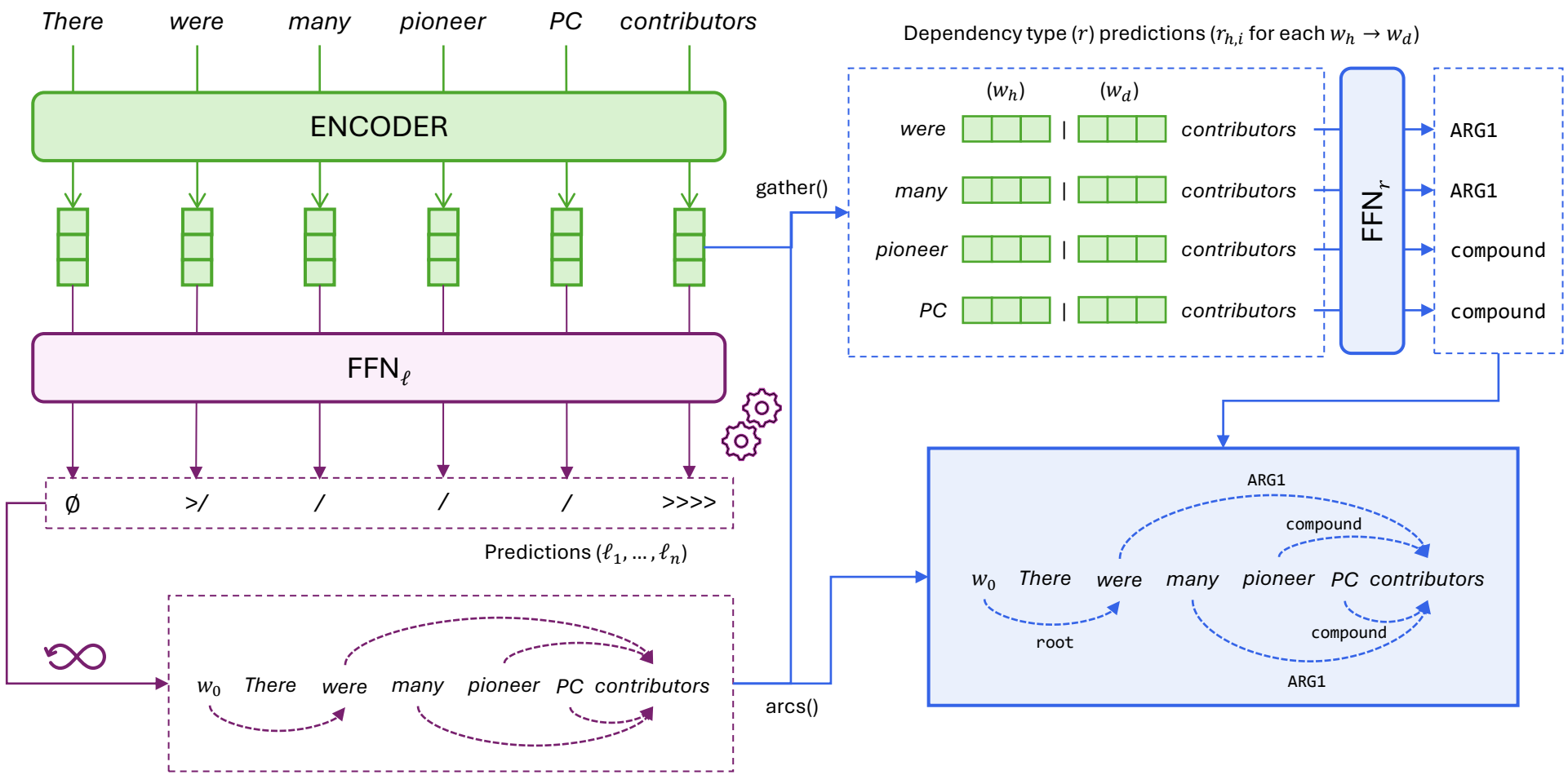


P1	000110	001001	111110	000000	111000	110001
P2	001000	000000	000000	100000	110000	000000
P3	000000	000000	000000	001000	110000	000000

Same as $4k$-bit encoding!

Encoder-Decoder Architecture



Experiments

Encoders:

- 4-layered BiLSTM.
- XLM-RoBERTa (Conneau et al., 2020).
- XLNet (Yang et al., 2019).

Decoders:

- Absolute (A) and relative (R).
- Bracketing (B) with $k \in \{2,3\}$.
- $4k$-bit (B4) with $k \in \{2,3,4\}$.
- $6k$-bit (B6) with $k \in \{2,3,4\}$.

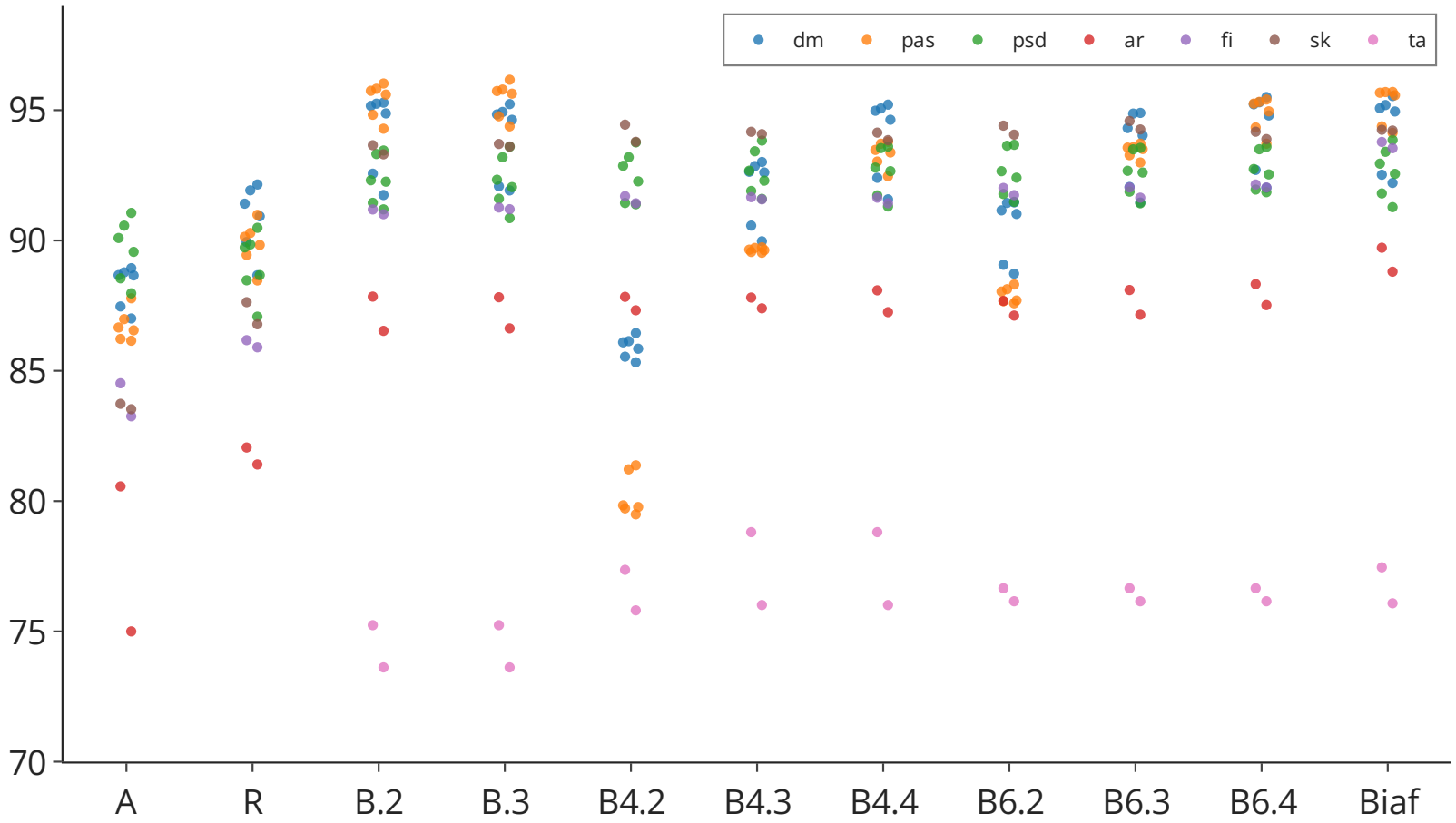
Evaluation

- SemEval 2015 Task 18 and IWPT-2021 Shared Task.
- UF and LF score.
- **Biaffine** baseline (Dozat & Manning, 2018).
- **Coverage** (score with gold labels).

Results

Performance

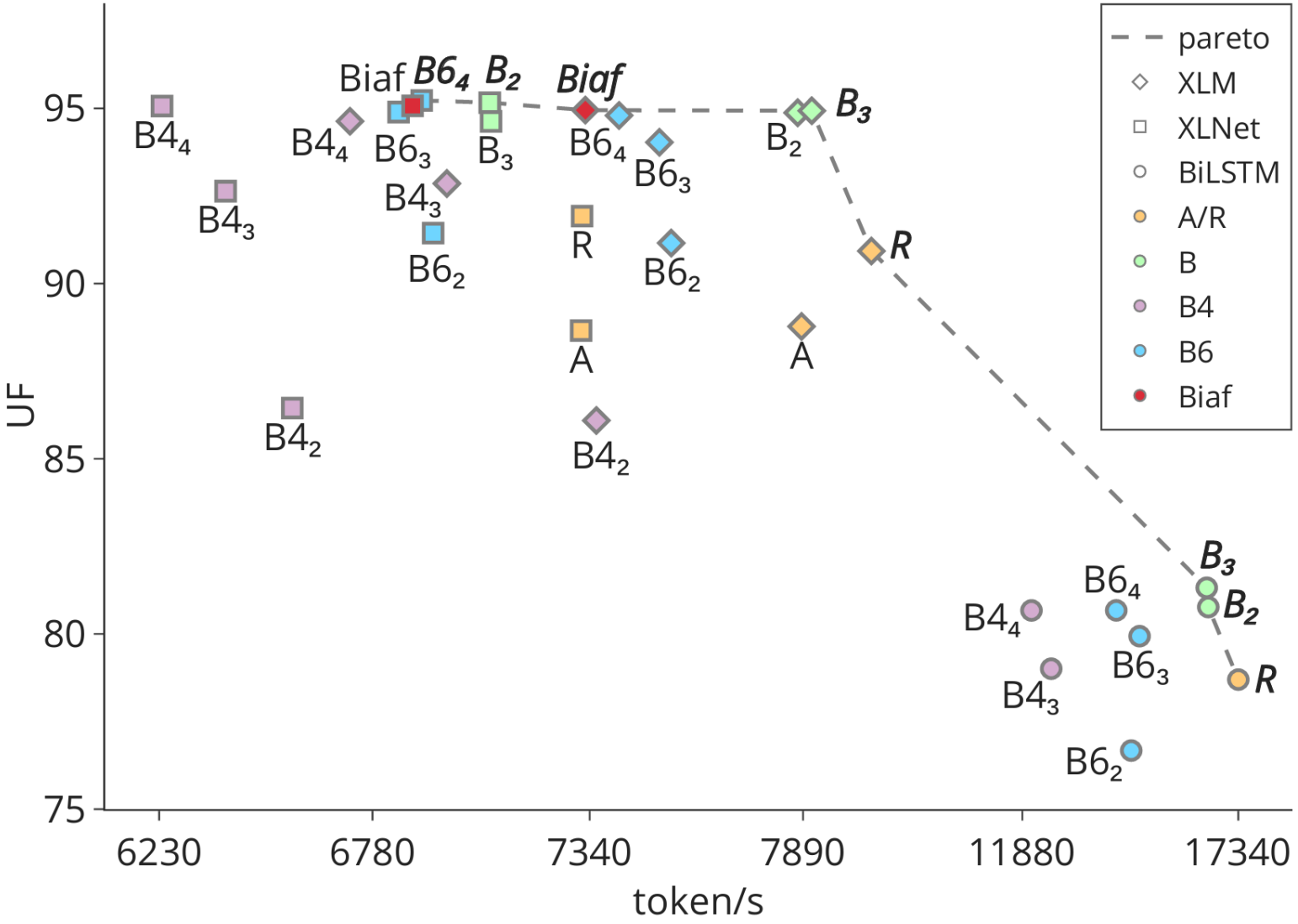
- Worst performance with **A** and **B**.
- **B2** and **B3** reach a similar performance to **Biaf**.
- Clear improvement increasing k.



Results

Speed

- Faster decoding for **A** and **R**.
- **B4.*k*** is slower than **Biaf** (likely due to the extra arcs generated).
- **B6.*k*** and **B.*k*** offer a trade-off between accuracy and speed.



Conclusions

- Several bounded and unbounded linearizations are proposed for graph parsing.
- Multilingual benchmark.
- SL algorithms and graph-based parsers have similar performance.
- SL is a faster and more efficient alternative to graph-based models.
- Hyperparameter k must be configured for bracket and bit encodings.

Thanks for listening!

Ana Ezquerro

David Vilares

Carlos Gómez-Rodríguez

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